$-x+4-3y+9=0 \longrightarrow -x-3y+13=0$

$4x-16-3z+15=0 \qquad 4x-3z-1=0$

Calcule la perpendicular común a las rectas: 1

$$r \equiv \frac{x-4}{3} = \frac{y-3}{-1} = \frac{z-5}{4}$$

$$s \equiv \begin{cases} 2x - z = 2 \\ y = -1 \end{cases} \equiv \begin{cases} x = \lambda \\ y = -1 \\ z = 2\lambda - 2 \end{cases} \quad \lambda \in \mathbb{R}$$

POSICIÓN RELATIVA

$\vec{v}(3,-1,4) \qquad \vec{w}(1,0,2)$

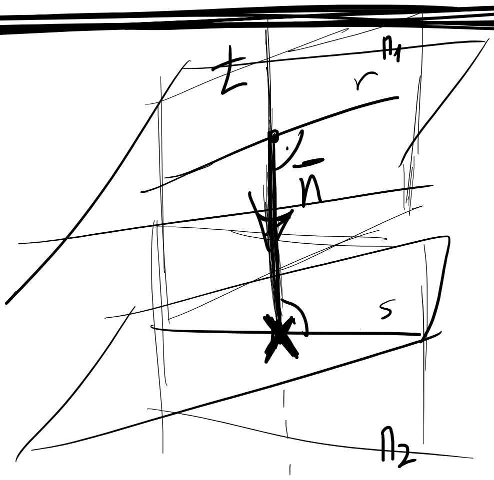
$$r \equiv \begin{cases} x = 4+3\mu \\ y = 3-1\mu \\ z = 5+4\mu \end{cases} \qquad s \equiv \begin{cases} 2x - z = 2 \\ y = -1 \end{cases}$$

$$r \cap s \equiv \begin{cases} x = 4+3\mu \\ y = 3-\mu \\ z = 5+4\mu \\ 2x - z = 2 \\ y = -1 \end{cases} \longrightarrow \begin{cases} 2(4+3\mu) - (5+4\mu) = 2 \\ 3-\mu = -1 \end{cases}$$

$\boxed{\mu = 4}$

$1 + 2\mu = 0 \longrightarrow \boxed{\mu = -1/2}$

$\Longrightarrow$ se cruzan.



1 MÉTODO:

$$\vec{n} = \vec{v} \times \vec{w} = \begin{vmatrix} i & j & k \\ 3 & -1 & 4 \\ 1 & 0 & 2 \end{vmatrix} = (-2, -2, 1)$$

$\pi_1 \equiv$ contiene a r $\to A(4,3,5)$, $\to \vec{v}(3,-1,4)$
dirección $\vec{n}$

$\pi_2 \equiv$ contiene a s $\to B(0,-1,-2)$, $\to \vec{w}(1,0,2)$
dirección $\vec{n}$

$$\pi_1 \equiv \begin{vmatrix} x-4 & y-3 & z-5 \\ 3 & -1 & 4 \\ -2 & -2 & 1 \end{vmatrix} = (x-4)\cdot 7 - (y-3)11 + (z-5)(-8) \Rightarrow$$

$-28 \quad +33 \quad +40$

$$\boxed{\pi_1 \equiv 7x - 11y - 8z + 45 = 0}$$

$$\pi_2 \equiv \begin{vmatrix} x & y+1 & z+2 \\ 1 & 0 & 2 \\ -2 & -2 & 1 \end{vmatrix} = x \cdot 4 - (y+1)\cdot 5 + (z+2)(-2) \Rightarrow$$

$$\boxed{\pi_2 \equiv 4x - 5y - 2z - 9 = 0}$$

$$\boxed{r \equiv \begin{cases} 7x - 11y - 8z + 45 = 0 \\ 4x - 5y - 2z - 9 = 0 \end{cases}}$$

MÉTODO 1'5:

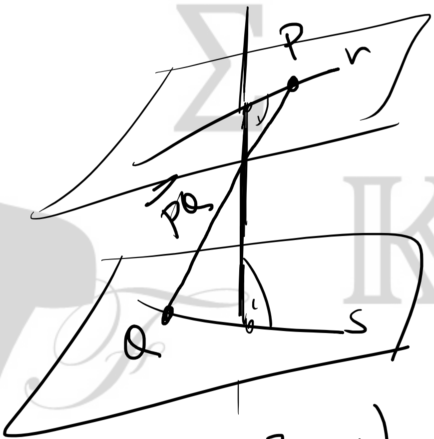
$$\pi_1 \cap s \equiv \begin{cases} 7x - 11y - 8z + 45 = 0 \\ x = \lambda \\ y = -1 \\ z = 2\lambda - 2 \end{cases} \longrightarrow \boxed{(8, -1, 14)}$$

$$7\lambda + 11 - 8(2\lambda - 2) + 45 = 0$$

$$-9\lambda + 72 = 0 \longrightarrow \boxed{\lambda = 8}$$

$$-9\lambda+72=0 \rightarrow \boxed{\lambda=8}$$

$$\boxed{t \equiv \begin{cases} x = 8-2\alpha \\ y = -1-2\alpha \\ z = 14+1\alpha \end{cases}, \quad \alpha \in \mathbb{R}}$$



MÉTODO 2:

$$\begin{cases} \overline{PQ}\cdot\overline{v}=0 \\ \overline{PQ}\cdot\overline{w}=0 \end{cases}$$

$\overline{PQ}(-2,-2,1)$

$P(10,1,13)$

$Q(8,-1,14)$

$P(4+3\mu, 3-\mu, 5+4\mu)$

$Q(\lambda, -1, 2\lambda-2)$

$\overline{PQ}(\lambda-4-3\mu, -1-3+\mu, 2\lambda-2-5-4\mu)$

$\overline{PQ}(\lambda-4-3\mu, -4+\mu, -7-4\mu+2\lambda)$

$\overline{v}(3,-1,4) \qquad \overline{w}(1,0,2)$

$$\begin{cases} \cancel{3\lambda}-12-\cancel{9\mu}+4-\cancel{\mu}-28-\cancel{16\mu}+\cancel{8\lambda}=0 \\ \cancel{\lambda}-4-\cancel{3\mu}-14-\cancel{8\mu}+\cancel{4\lambda}=0 \end{cases}$$

$$\cancel{\lambda} - 4 - 3\cancel{\mu} - 14 - 8\cancel{\mu} + \cancel{12}\lambda = 0$$

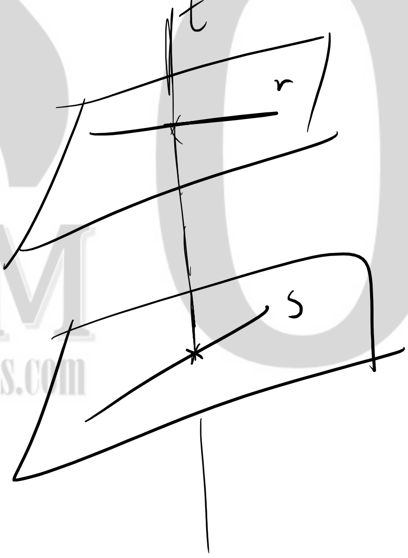
$$\left.\begin{array}{l} 11\lambda - 26\mu - 36 = 0 \\ 5\lambda - 11\mu - 18 = 0 \end{array}\right\} \quad \begin{array}{l} \lambda = \dfrac{26\mu + 36}{11} \\ \lambda = \dfrac{11\mu + 18}{5} \end{array}$$

$$\frac{26\mu + 36}{11} = \frac{11\mu + 18}{5}$$

$$130\mu + 180 = 121\mu + 198$$

$$9\mu = 18 \longrightarrow \boxed{\mu = 2} \quad \boxed{\lambda = 8}$$

$$\boxed{t \equiv \left\{\begin{array}{l} x = 10 - 2\beta \\ y = 1 - 2\beta \\ z = 13 + 1\beta \end{array}\right. \quad , \ \beta \in \mathbb{R}}$$



MÉTODO 3:

$$r \equiv \left\{\begin{array}{l} -x - 3y + 13 = 0 \\ 4x - 3z - 1 = 0 \end{array}\right.$$

$$s \equiv \left\{\begin{array}{l} 2x - z - 2 = 0 \\ y + 1 = 0 \end{array}\right.$$

$$h_1 \equiv -x - 3y + 13 + K \cdot (4x - 3z - 1) = 0$$

$$\pi_2 \equiv 2x - z - 2 + m\cdot(y+1) = 0$$

$$\pi_1 \equiv (-1+4k)x - 3y - 3kz + 13 - k = 0$$

$$\pi_2 \equiv 2x + my - z - 2 + m = 0$$

$$r \equiv \begin{cases} (-1+4k)x - 3y - 3kz + 13 - k = 0 \\ 2x + my - z - 2 + m = 0 \end{cases}$$

$$\vec{n} = \begin{vmatrix} i & j & k \\ 4k-1 & -3 & -3k \\ 2 & m & -1 \end{vmatrix} =$$

$$= (3+3km,\ 4k-1-6k,\ 4km-m+6)$$

$$= (3+3km,\ -1-2k,\ 4km-m+6)$$

$$\vec{v}(3,-1,4) \qquad \vec{w}(1,0,2)$$

$$\left.\begin{aligned} &\cancel{9} + 9km + \cancel{1} + 2k + 16km - 4m + \cancel{24} = 0 \\ &\cancel{3} + 3km + 8km - 2m + \cancel{12} = 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} &34 + 2k - 4m + 25km = 0 \\ &15 - 2m + 11km = 0 \end{aligned}\right\}$$

$$\boxed{k = \frac{2m-15}{11m}}$$

1

$$34 + 2\left(\frac{2m-15}{11m}\right) - 4m + 25\left(\frac{2m-15}{11\cancel{m}}\right)\cancel{m} = 0$$

$$37\cancel{4m} + \cancel{4m} - 30 - \cancel{44m^2} + \cancel{50m^2} - \cancel{375}m = 0$$

$$6m^2 + 3m - 30 = 0$$

$$2m^2 + m - 10 = 0 \quad \begin{cases} \to m = 2 \to k = -\frac{1}{2} \\ \to m = -\frac{5}{2} \to k = \frac{8}{11} \end{cases}$$

$$\cancel{m = 2,\ k = -1/2} \to \vec{n}(0, 0, 0)$$

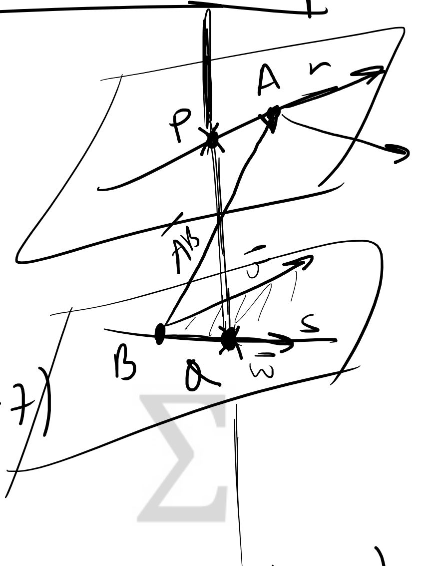
$$m = -\frac{5}{2},\ k = \frac{8}{11} \to \vec{n}\left(-\frac{27}{11}, -\frac{27}{11}, \frac{27}{22}\right) \parallel (-2, -2, 1)$$

$$t \equiv \begin{cases} (-1+4k)x - 3y - 3kz + 13 - k = 0 \\ 2x + my - z - 2 + m = 0 \end{cases}$$

$$t \equiv \begin{cases} \frac{21}{11}x - 3y - \frac{24}{11}z + \frac{135}{11} = 0 \\ 2x - \frac{5}{2}y - z - \frac{9}{2} = 0 \end{cases}$$

$$t \equiv \begin{cases} 21x - 33y - 24z + 135 = 0 \\ 4x - 5y - 2z - 9 = 0 \end{cases}$$

$$t \equiv \left\{ 4x - 5y - 2z - 9 = 0 \right.$$



MÉTODO 4:

$$d(P, Q) = d(r, s)$$

$A(4, 3, 5)$, $B(0, -1, -2)$ $\Big\}$ $\overline{AB} = (-4, -4, -7)$

$P(4+3\mu, 3-\mu, 5+4\mu)$

$$|\overline{PQ}| = d(r, s)$$

$Q(\lambda, -1, 2\lambda - 2)$

$$\overline{PQ}(\lambda - 4 - 3\mu, -4+\mu, -7-4\mu+2\lambda)$$

(*)$_5$

$$d(r, s) = \frac{|[\overline{AB}, \vec{v}, \vec{w}]|}{|\vec{v} \times \vec{w}|} = \frac{\overline{AB} \cdot (\vec{v} \times \vec{w})}{|\vec{v} \times \vec{w}|} = 3$$

$$\vec{n} = \vec{v} \times \vec{w} = (-2, -2, 1) \qquad |\vec{n}| = \sqrt{9} = 3$$

$$(-4, -4, -7)(-2, -2, 1) = 8 + 8 - 7 = 9$$

$$|(\lambda - 4 - 3\mu, -4+\mu, -7-4\mu+2\lambda)| = 3$$

$$\sqrt{\quad (\quad)^2 + (\quad)^2 + (-7-4\mu+2\lambda)^2} = 3$$

$$\sqrt{(\lambda-4-3\mu)^2+(-4+\mu)^2+(-7-4\mu+2\lambda)^2}=3$$

$$\lambda^2-4\lambda-3\mu\lambda-4\lambda+16+12\mu-3\mu\lambda+12\mu+9\mu^2$$

$$+16-8\mu+\mu^2+$$

$$+4\lambda^2-14\lambda-8\lambda\mu-14\lambda+49+28\mu-8\lambda\mu+28\mu+16\mu^2=$$

$$5\lambda^2-18\lambda-11\lambda\mu-18\lambda+81+40\mu-11\lambda\mu+32\mu+26\mu^2=$$

$$26\mu^2+5\lambda^2-22\lambda\mu-36\lambda+72\mu+81=9$$

$$26\mu^2+5\lambda^2-22\lambda\mu-36\lambda+72\mu+72=0$$

$$26\mu^2+(72-22\lambda)\mu+5\lambda^2-36\lambda+72=0$$

$$\mu=\frac{22\lambda-72\pm\sqrt{(72-22\lambda)^2-104(5\lambda^2-36\lambda+72)}}{52}$$

$$\Delta=5184+484\lambda^2-3168\lambda-520\lambda^2+3744\lambda-7488=$$

$$=-36\lambda^2+576\lambda-2304=$$

$$=-36(\lambda^2-16\lambda+64)=-36(\lambda-8)^2$$

$$\boxed{\lambda=8}\longrightarrow\mu=\frac{104}{52}=2\to\boxed{\mu=2}$$

$$P(4+3\mu,\ 3-\mu,\ 5+4\mu)\xrightarrow{\mu=2}(10,1,13)$$

$P(4+3\mu, 3-\mu, 5+4\mu) \xrightarrow{\mu=2} (10, 1, 13)$

$Q(\lambda, -1, 2\lambda-2) \xrightarrow{\lambda=8} (8, -1, 14)$

$\overrightarrow{PQ}\ (-2, -2, 1)$

$$\boxed{t \equiv \begin{cases} x = 8 - 2\gamma \\ y = -1 - 2\gamma \\ z = 14 + 1\gamma \end{cases} \quad \gamma \in \mathbb{R}}$$

MÉTODO 5:

$$d^2(P,Q) = (\lambda-4-3\mu)^2 + (-4+\mu)^2 + (-7-4\mu+2\lambda)^2$$

$$\min d(P,Q) \equiv \min d^2(P,Q)$$

$$\frac{\partial d^2}{\partial \lambda} = 2(\lambda-4-3\mu) + 2(-7-4\mu+2\lambda)\cdot 2$$

$$\frac{\partial d^2}{\partial \mu} = 2(\lambda-4-3\mu)(-3) + 2(-4+\mu) + 2(-7-4\mu+2\lambda)(-4)$$

$$24 - 6\lambda + 18\mu - 8 + 2\mu + 56 + 32\mu - 16\lambda$$

$$\left.\begin{array}{l} 10\lambda - 22\mu - 36 = 0 \\ -22\lambda + 52\mu + 72 = 0 \end{array}\right\}$$

$$-22\lambda+52\mu+72=0$$

$$\left.\begin{aligned}5\lambda-11\mu&=18\\-11\lambda+26\mu&=-36\end{aligned}\right\}\quad \begin{aligned}\lambda&=\frac{18+11\mu}{5}\\ \lambda&=\frac{26\mu+36}{11}\end{aligned}$$

$$\frac{18+11\mu}{5}=\frac{26\mu+36}{11}$$

$$198+121\mu=130\mu+180$$

$$18=9\mu\rightarrow\boxed{\mu=2}$$

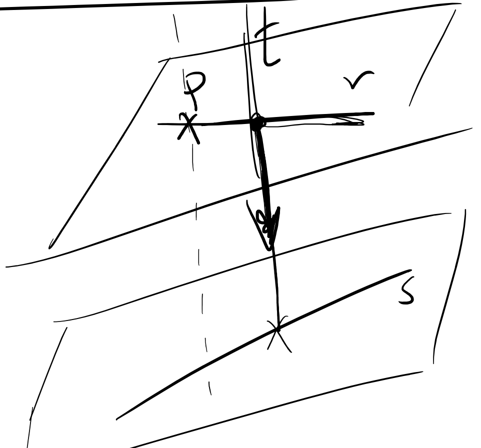
$$\boxed{\lambda=8}$$

$$P(4+3\mu,\ 3-\mu,\ 5+4\mu)\xrightarrow{\mu=2}(10,1,13)$$

$$Q(\lambda,\ -1,\ 2\lambda-2)\xrightarrow{\lambda=8}(8,-1,14)$$

$$\overline{PQ}\ (-2,-2,1)$$

$$\boxed{t\equiv\begin{cases}x=8-2\gamma\\y=-1-2\gamma\\z=14+1\gamma\end{cases}\quad\gamma\in\mathbb{R}}$$



MÉTODO 6:

$$\overline{n}\ (-2,-2,1)$$

$$P(4+3\mu,\ 3-\mu,\ 5+4\mu)$$

$P(4+3\mu,\ 3-\mu,\ 5+4\mu)$

$$s \equiv \begin{cases} 2x - z = 2 \\ y = -1 \end{cases}$$

$$\mu \in \mathbb{R}$$

$$t_\mu \equiv \begin{cases} x = \overbrace{4+3\mu}^{P} \overbrace{-2\alpha}^{\vec{n}} \\ y = 3-\mu -2\alpha \\ z = 5+4\mu + \alpha \end{cases}, \quad \alpha \in \mathbb{R}$$

$$\begin{cases} 2(4+3\mu-2\alpha) - (5+4\mu+\alpha) = 2 \\ 3-\mu-2\alpha = -1 \end{cases}$$

$$\begin{cases} 2\mu - 5\alpha = -1 \\ -\mu - 2\alpha = -4 \end{cases} \qquad \boxed{\mu = 4-2\alpha}$$

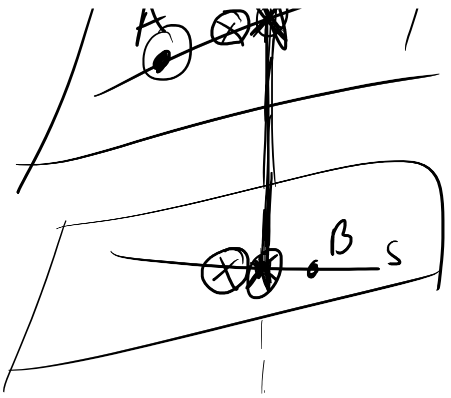
$$8 - 4\alpha - 5\alpha = -1 \rightarrow -9\alpha = -9$$

$$\boxed{\alpha = 1}$$

$$\boxed{\mu = 2}$$

$$t \equiv \begin{cases} x = 10 - 2\alpha \\ y = 1 - 2\alpha \\ z = 13 + \alpha \end{cases}, \quad \alpha \in \mathbb{R}$$

MÉTODO 7:



$P(4+3\mu, 3-\mu, 5+4\mu) \longrightarrow A(4,3,5)$

$Q(\lambda, -1, 2\lambda-2) \longrightarrow B(0,-1,-2)$

1 A, Q

$\overrightarrow{AQ} = (\lambda-4, -4, 2\lambda-7)$

$|\overrightarrow{AQ}|^2 = (\lambda-4)^2 + 16 + (2\lambda-7)^2 =$

$= \lambda^2 + 16 - 8\lambda + 16 + 4\lambda^2 + 49 - 28\lambda =$

$= 5\lambda^2 - 36\lambda + 81$

$\left(|\overrightarrow{AQ}|^2\right)' = 10\lambda - 36 \longrightarrow \boxed{\lambda = 3'6}$

$\lambda = 3'6 \longrightarrow Q(3'6, -1, 5'2)$

$P(4+3\mu, 3-\mu, 5+4\mu)$

2 $|\overrightarrow{PQ}|^2 = (0'4+3\mu)^2 + (4-\mu)^2 + (-0'2+4\mu)^2$

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$$= 0'16 + \cancel{9\mu^2} + \cancel{2'4\mu} + 16 + \cancel{\mu^2} - \cancel{8\mu}$$

$$+ 0'04 + \cancel{16\mu^2} - \cancel{1'6\mu} =$$

$$= 26\mu^2 - 7'2\mu + 16'2 \longrightarrow ¡\mu!$$

$P(\quad , \quad , \quad)$

$Q(\lambda, -1, 2\lambda - 2)$

3 . . .

<u>MÉTODO 7'5:</u>