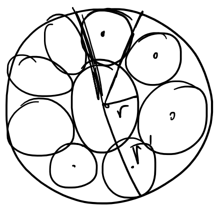
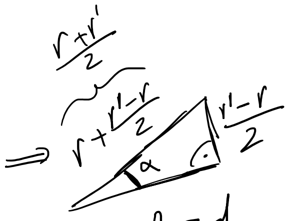
1. Sean C y C' dos circunferencias concéntricas de radios r y r' respectivamente, con $r < r'$. En la corona limitada por C y C' existen ocho circunferencias C_i, con tangentes a C y C' y de tal modo que C_i es tangente a C_{i+1} para $i = 1, 2, ..., 7$, y C_8 es también tangente a C_1. Determinar valor de $\frac{r'}{r}$.



$$¿\frac{r'}{r}?$$

$$\frac{360°}{8} = \frac{180}{4} = 45° \quad \frac{\pi}{4} \text{ rad}$$



$$\alpha = \frac{\pi}{8} \text{ rad}$$

$$\operatorname{sen}\frac{\pi}{8} = \sqrt{\frac{1-\cos\pi/4}{2}} = \sqrt{\frac{1-\sqrt{2}/2}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

$$\operatorname{sen}\frac{\pi}{8} = \frac{(r'-r)/\cancel{2}}{(r+r')/\cancel{2}} = \frac{r'-r}{r+r'}$$

$$\boxed{\frac{\sqrt{2-\sqrt{2}}}{2} = \frac{r'-r}{r+r'}}$$

$$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$$

$$\operatorname{sen}^2\alpha + \cos^2\alpha = 1$$

$$\cos^2\alpha = 1 - \operatorname{sen}^2\alpha$$

$$\cos 2\alpha = 1 - 2\operatorname{sen}^2\alpha$$

$$\frac{1-\cos 2\alpha}{2} = \operatorname{sen}^2\alpha$$

$$\operatorname{sen}\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}}$$

$$\left(\sqrt{2-\sqrt{2}}\right)(r+r') = 2r' - 2r$$

$$\sqrt{2-\sqrt{2}}\, r + \sqrt{2-\sqrt{2}}\, r' = 2r' - 2r$$

$$\left(\sqrt{2-\sqrt{2}} - 2\right) r' = \left(-\sqrt{2-\sqrt{2}} - 2\right) r$$

$$\boxed{\frac{r'}{r} = \frac{+\sqrt{2-\sqrt{2}} + 2}{-\sqrt{2-\sqrt{2}} + 2}}$$