PAIS VASCO - 2020 - MII

$\boxed{A1}$ $$(A|B) = \left(\begin{array}{ccc|c} a & -1 & 2 & 2 \\ 1 & -2 & -1 & 1 \\ 1 & 2 & a & 3 \end{array}\right)$$

$$|A| = -2a^2+4+1+4+a+2a = -2a^2+3a+9$$

$$|A|=0 \Longleftrightarrow \cancel{a^2+a+3=0}$$
$$-2a^2+3a+9=0 \to a=3,\quad a=-3/2$$

Si $a \neq 3, -3/2 \to |A| \neq 0 \to$ $\left.\begin{array}{l} rg(A)=3 \\ rg(A|B)=3 \\ \text{nº incog}=3 \end{array}\right\}$ S.C.D.

$$|A_x| = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -1 \\ 3 & 2 & a \end{vmatrix} = -4a+4+3+12+a+4 = -3a+23$$

$$|A_y| = \begin{vmatrix} a & 2 & 2 \\ 1 & 1 & -1 \\ 1 & 3 & a \end{vmatrix} = a^2+6-2-2-2a+3a = a^2+a+2$$

$$|A_z| = \begin{vmatrix} a & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix} = -6a+4-1+4+3-2a = -8a+10$$

$$x=\frac{-3a+23}{-2a^2+3a+9}$$

$$y=\frac{a^2+a+2}{-2a^2+3a+9}$$

$$z=\frac{-8a+10}{-2a^2+3a+9}$$

Si $a=3 \rightarrow |A|=0$

$$(A|B)=\left(\begin{array}{ccc|c} 3 & -1 & 2 & 2 \\ 1 & -2 & -1 & 1 \\ 1 & 2 & 3 & 3 \end{array}\right) \rightarrow \begin{vmatrix} -1 & 2 \\ -2 & -1 \end{vmatrix} = 5 \neq 0 \Rightarrow rg(A)=2$$

$$\begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -1 \\ 3 & 2 & 3 \end{vmatrix} = \cancel{-12}+4+3+\cancel{12}+3+4 = 14 \neq 0 \rightarrow rg(A|B)=3$$

<u>S.I</u>

2a Si $a = -3/2 \longrightarrow |A| = 0$

$$(A|B) = \left(\begin{array}{ccc|c} -3/2 & -1 & 2 & 2 \\ 1 & -2 & -1 & 1 \\ 1 & 2 & -3/2 & 3 \end{array}\right)$$

$$\begin{vmatrix} -1 & 2 \\ -2 & -1 \end{vmatrix} \neq 0 \Downarrow rg(A) = 2$$

$$\begin{vmatrix} -1 & 2 & 2 \\ -2 & -1 & 1 \\ 2 & -3/2 & 3 \end{vmatrix} = 3 + 6 + 4 + 4 + 12 - \frac{3}{2} \neq 0$$

$\rightarrow rg(A|B) = 3$

$\rightarrow$ **S.I**

$\boxed{B1}$ $M = \begin{pmatrix} 1 & \alpha & 1 \\ \alpha & 1 & \alpha \\ 0 & \alpha & 1 \end{pmatrix}$

(a) $|M| = 1 + \alpha^2 - \alpha^2 - \alpha^2 = 1 - \alpha^2$

$|M| = 0 \Leftrightarrow 1 - \alpha^2 = 0 \begin{cases} \alpha = 1 \\ \alpha = -1 \end{cases}$

Si $\alpha \neq \pm 1 \rightarrow |M| \neq 0 \rightarrow M$ invertible.

(b) $M(0) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $|M| = 1$

$$M = \frac{1}{|M|}\left(Adj\ M\right)^t$$

$$Adj\ M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \Rightarrow M^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\boxed{A2}$ $A(-1,2,3)$

(a) $\vec{v}(-1,-2,-3)$ $\pi \equiv \begin{vmatrix} x+1 & y-2 & z-3 \\ -1 & -2 & -3 \\ 1 & 3 & 5 \end{vmatrix} =$

$\vec{w}(1,3,5)$

$$= (x+1)(-1) - (y-2)(-2) + (z-3)(-1) =$$

$$= -x-1+2y-4-z+3 \Rightarrow$$

$$\boxed{\pi \equiv -x+2y-z-2=0}$$

b) ¿A?

$\pi \equiv -x+2y-z-2=0$

$\pi' \equiv Ax-y+5z=8$

$\pi \perp \pi'$ $(-1,+2,-1)\cdot(A,-1,5)=0$

$-A-2-5=0 \to \boxed{A=-7}$

B2)

$\pi \equiv 2x-y+Az=0$

$r \equiv \begin{cases} 4x-3y+4z=-1 \\ 3x-2y+z=-3 \end{cases} \equiv \begin{cases} x=-2+5\lambda \\ y=-1+8\lambda \\ z=1+\lambda \end{cases}$

$r \parallel \pi$ $\vec{n}_\pi(2,-1,A)$

$\vec{v}_r = \begin{vmatrix} i & j & k \\ 4 & -3 & 4 \\ 3 & -2 & 1 \end{vmatrix} = (5,8,1)$

$z=-3-3x+2y \to 4x-3y+4(-3-3x+2y)=-1$

$4x-3y-12-12x+8y=-1$

$-8x+5y=11$ $\quad x=-2$

$5y=-5 \to y=-1$

$z=-3+6-2=1$

$\hookrightarrow A(-2,-1,1)$

$r \parallel \pi$ si $\vec{v_r} \perp \vec{n_\pi} \leftrightarrow \vec{v_r} \cdot \vec{n_\pi} = 0$

$(5,8,1)\cdot(2,-1,A) = 0 \quad 10-8+A=0 \leftrightarrow \boxed{A=-2}$

- $\pi_2 \perp r \quad (0,0,0) \in \pi_2$

$\vec{n_{\pi_2}} \parallel v_r \ (5,8,1)$

$5x+8y+z+D=0 \rightarrow D=0$

$\boxed{\pi_2 \equiv 5x+8y+z=0}$

$\boxed{A3}$ $f(x) = ax^3+bx^2+c \rightarrow f'(x) = 3ax^2+2bx$

$\cancel{f(0)}$ $f(0)=2 \rightarrow \boxed{c=2}$

$f(1)=-1 \rightarrow a+b+2=-1 \rightarrow a+b=-3$

$f'(1)=0 \rightarrow 3a+2b=0$

$\left.\begin{array}{l} a+b=-3 \\ 3a+2b=0 \end{array}\right| \rightarrow b=\frac{-3a}{2}$

$\rightarrow a-\frac{3a}{2}=-3 \rightarrow \frac{-a}{2}=-3 \quad \boxed{a=+6}$

$\boxed{b=-9}$

$f'(x)=18x^2-18x \longrightarrow f''(x)=36x-18$

$f'(x)=0 \iff 18x(x-1)=0$ → $x=0$; → $x=1$

Si $x=0 \to f''(0)=-18<0 \to$ $\boxed{x=0 \text{ máximo local}}$

$\boxed{B3}$ $f(x)=x^2+9$ $\quad P(0,0)$

$f'(x)=2x$

$y-f(a)=f'(a)(x-a)$

$y-(a^2+9)=2a(x-a)$

$-(a^2+9)=2a(-a)$

$-a^2-9=-2a^2$

$-9=-a^2 \longrightarrow a^2=9 \longrightarrow \boxed{a=\pm 3}$

$\longrightarrow y-18=6(x-3)$

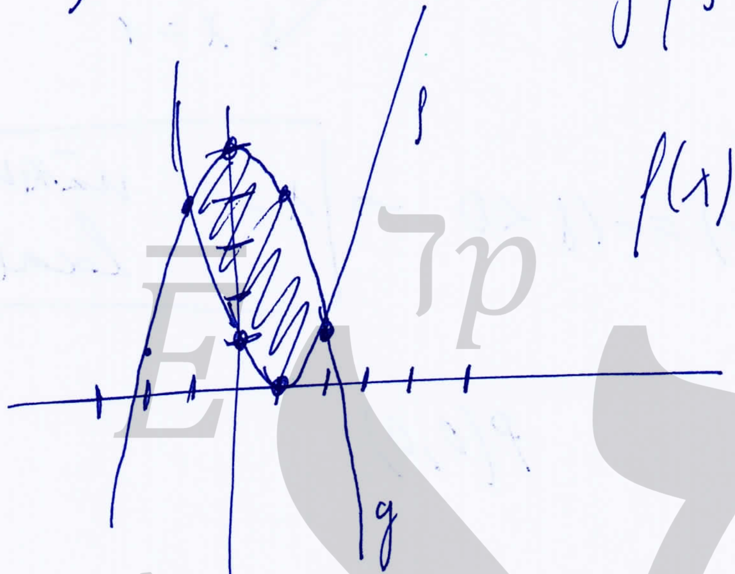
$\longrightarrow y-18=-6(x+3)$

$\boxed{A4}$ $f(x) = x^2 - 2x + 1$

x	0	1	2	-2	-1
y	1	0	1	9	4

$g(x) = -x^2 + 5$

x	0	1	-1	2	-2
y	5	4	4	1	1



$f(x) = g(x) \Rightarrow \begin{cases} x = -1 \\ x = 2 \end{cases}$

$$A = \left| \int_{-1}^{2} \left[(x^2 - 2x + 1) - (-x^2 + 5) \right] dx \right| =$$

$$= \left| \int_{-1}^{2} (2x^2 - 2x - 4)\, dx \right| = \left| \frac{2x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}} - 4x \right]_{-1}^{2} =$$

$$= \left| \frac{16}{3} - 4 - 8 - \left(-\frac{2}{3} - 1 + 4 \right) \right| =$$

$$= \left| \frac{16}{3} - 12 + \frac{2}{3} - 3 \right| = 9 \ u^2$$

$\boxed{B4}$ $I = \int x \cos(2x)\,dx = \left[\begin{array}{ll} u = x & du = dx \\ dv = \cos(2x)dx & v = \frac{1}{2}\,\text{sen}(2x)dx \end{array}\right]$

$= \frac{x}{2}\,\text{sen}\,2x - \int \frac{1}{2}\,\text{sen}(2x)\,dx =$

$= \frac{x}{2}\,\text{sen}\,2x + \frac{1}{2}\cdot\frac{1}{2}\cos(2x) + C$

$J = \int \frac{dx}{x^2+2x-3} =$

$\frac{1}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$

$x^2+2x-3=0$

$x=1$ $(x-1)(x+3)$

$x=-3$

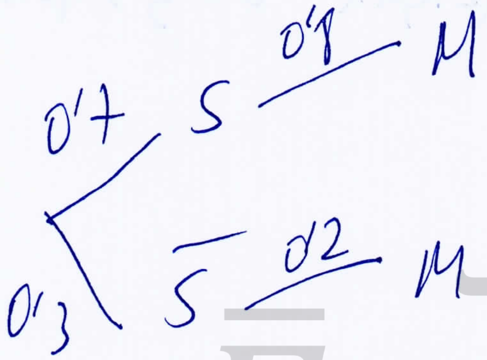
$x=1 \rightarrow 1 = 4A \rightarrow A = 1/4$

$x=-3 \rightarrow 1 = -4B \rightarrow B = -1/4$

$= \int \frac{1/4}{x-1}dx + \int \frac{-1/4}{x+3}dx = \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+3| + C$

$= \frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C.$

A5 $S \equiv$ "satisfecho" $M \equiv$ "más de 1000€"



ⓐ $p(M) = 0'7\cdot 0'8 + 0'3\cdot 0'2 = 0'62$

ⓑ $p(S|M) = \dfrac{p(S\cap M)}{p(M)} = \dfrac{0'7\cdot 0'8}{0'62} = 0'9032$

ⓒ $p(\overline{M}\cap S) = 0'7\cdot 0'2 = 0'14$

B5 $n=30$ $p=0'4$

ⓐ $X \equiv$ "# aparcamientos ocupados de entre 30"

$X \sim B(30, 0'4) \xrightarrow[n\cdot q>5]{n\cdot p>5} N(12, 2'68)$

ⓑ $p(X=8) = \binom{30}{8} 0'4^8 \cdot 0'6^{22} = 0'0505$

ⓒ $p(10 \leq X \leq 20) = p(9'5 < X < 20'5) =$

10

$$= P\left(\frac{9'5-12}{2'68} < Z < \frac{20'5-12}{2'68}\right) =$$

$$= P(-0'93 < Z < 3'17) =$$

$$= P(Z < 3'17) - P(Z < -0'93) =$$

$$= P(Z < 3'17) - (1 - P(Z < 0'93)) =$$

$$= 0'9992 - 1 + 0'8238 = 0'823$$