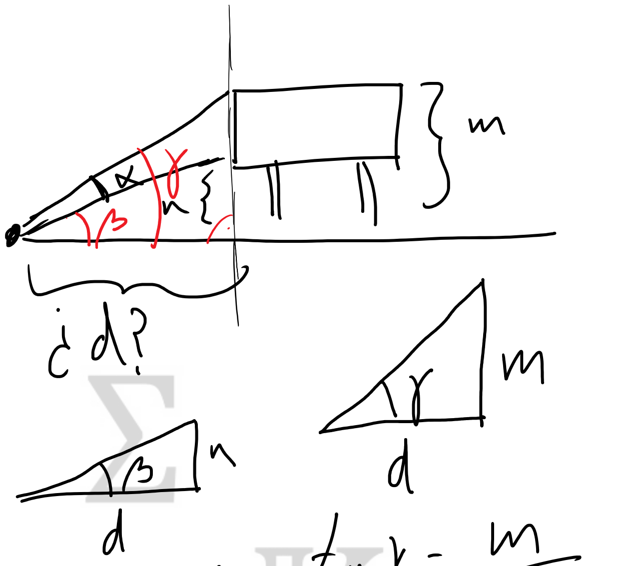
Un cartel, situado en una pared, tiene sus bordes superior e inferior a alturas m y n, respectivamente, con referencia a la visual de un lector. ¿A qué distancia de la pared debe colocarse el lector del cartel para que el ángulo visual determinado por la pupila y los bordes sea máximo?



$\operatorname{tg} \beta = \frac{n}{d}$ $\quad$ $\operatorname{tg} \gamma = \frac{m}{d}$

$$\alpha = \gamma - \beta =$$

$$= \operatorname{atan} \frac{m}{d} - \operatorname{atan} \frac{n}{d}$$

$$\alpha' = \frac{d\alpha}{dd} =$$

$$= \frac{-m/d^2}{1+\left(\frac{m}{d}\right)^2} - \frac{-n/d^2}{1+\left(\frac{n}{d}\right)^2} =$$

$$= \frac{-m/\cancel{d^2}}{\frac{d^2+m^2}{\cancel{d^2}}} + \frac{n/\cancel{d^2}}{\frac{d^2+n^2}{\cancel{d^2}}} = \frac{-m}{d^2+m^2} + \frac{n}{d^2+n^2}$$

$$\alpha' = 0 \Longleftrightarrow \frac{m}{d^2+m^2} = \frac{n}{d^2+n^2} \Longleftrightarrow$$

$$m(d^2+n^2) = n(d^2+m^2) \Longleftrightarrow$$

$$md^2 + mn^2 = nd^2 + nm^2 \Longleftrightarrow$$

$$md^2 - nd^2 = nm^2 - mn^2 \Longleftrightarrow$$

$$d^2 \cancel{(m-n)} = nm \cancel{(m-n)} \overset{m \neq n}{\Longleftarrow}$$

$$d^2 \cancel{(m-n)} = nm \cancel{(m-n)} \Longleftrightarrow$$

$$d^2 = nm \Longleftrightarrow \boxed{d = \sqrt{nm}}$$

$$\alpha' = \frac{-m}{d^2+m^2} + \frac{n}{d^2+n^2}$$

$$\alpha'' = \frac{+m \cdot 2d}{(d^2+m^2)^2} + \frac{-n2d}{(d^2+n^2)^2} = 2d\left[\frac{m}{(d^2+m^2)^2} - \frac{n}{(d^2+n^2)^2}\right]$$

$$\alpha''(\sqrt{nm}) = 2\sqrt{nm}\left[\frac{m}{(nm+m^2)^2} - \frac{n}{(nm+n^2)^2}\right] =$$

$$= 2\sqrt{nm}\left[\frac{\cancel{m}}{m^{\cancel{2}}(n+m)^2} - \frac{\cancel{n}}{n^{\cancel{2}}(m+n)^2}\right] =$$

$$= \frac{\overset{>0}{2\sqrt{nm}}}{\underset{>0}{(n+m)^2}}\underbrace{\left[\frac{1}{m} - \frac{1}{n}\right]}_{<0} < 0$$

$$\sqrt{nm} > 0 \qquad m > n \longrightarrow \frac{1}{m} < \frac{1}{n} \longrightarrow$$

$$(n+m)^2 > 0 \qquad \longrightarrow \frac{1}{m} - \frac{1}{n} < 0$$

$$\alpha''(\sqrt{nm}) < 0 \longrightarrow \boxed{d = \sqrt{nm} \text{ máximo relativo}}$$

α (1 mm)

relativo
para la
distancia