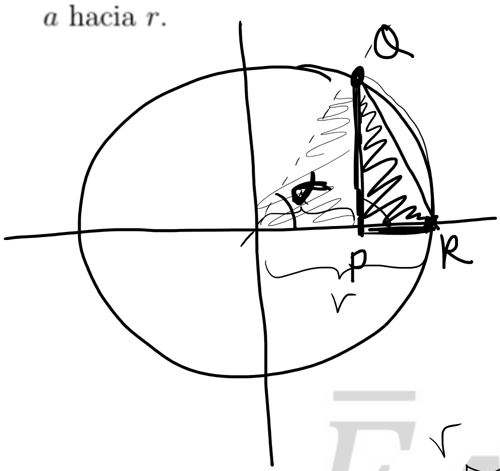
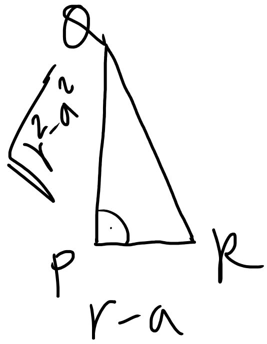
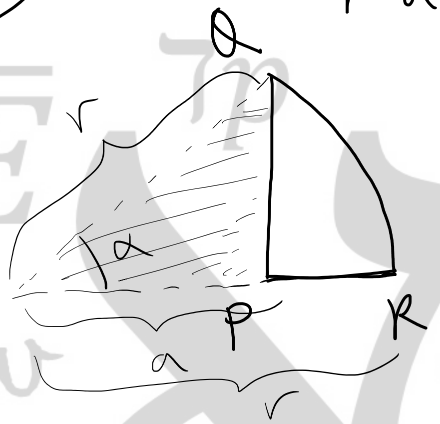
Tomando sobre el eje OX un punto $P(a,0)$, construímos sobre la circunferencia $x^2+y^2=r^2$ $(0 \leq a < r)$ el triángulo de vértices $P(a,0)$, $R(r,0)$, $Q(a,\sqrt{r^2-a^2})$. Consideremos ahora el triángulo curvilíneo cuyos lados son: el segmento $\overline{PQ}$, el segmento $\overline{PR}$, y el arco de circunferencia QR.

Calcular el límite del cociente de las áreas de los triángulos mencionados si hacemos tender a hacia r.





$$A_1 = \frac{(r-a)\sqrt{r^2-a^2}}{2}$$



$$A_2 = \frac{\cancel{\pi}\cdot r^2\cdot\alpha}{2\cancel{\pi}} - \frac{a\sqrt{r^2-a^2}}{2}$$

$$\cos\alpha = \frac{a}{r}$$

(*) $\alpha = \arccos\frac{a}{r}$

$$\lim_{a\to r}\frac{A_2}{A_1} = \lim_{a\to r}\frac{\frac{r^2\cdot\arccos\frac{a}{r}}{\cancel{2}} - \frac{a}{\cancel{2}}\sqrt{r^2-a^2}}{\frac{r-a}{\cancel{2}}\sqrt{r^2-a^2}} =$$

$$= \lim_{a\to r}\frac{r^2\arccos\frac{a}{r} - a\sqrt{r^2-a^2}}{(r-a)\sqrt{r^2-a^2}} = \left[\frac{0}{0}\right] =$$

L'Hopital

$$= \lim \left[r^2\cdot\frac{-1/r}{\sqrt{1-\left(\frac{a}{r}\right)^2}}^{*} - \sqrt{r^2-a^2} - a\,\frac{-2a}{\cancel{2}\sqrt{r^2-a^2}} \right]\cdot\sqrt{r^2-a^2} =$$

$$= \lim_{a\to r} \frac{\left[r \cdot \frac{\ldots}{\sqrt{1-\left(\frac{a}{r}\right)^2}} \ldots \frac{\ldots}{\cancel{2}\sqrt{r^2-a^2}} \right]}{\left[-\sqrt{r^2-a^2} + (r-a)\frac{-\cancel{2}a}{\cancel{2}\sqrt{r^2-a^2}} \right]\cdot\sqrt{r^2-a^2}} =$$

$$* \quad \frac{-1/r}{\sqrt{1-\left(\frac{a}{r}\right)^2}} = \frac{-1/\cancel{r}}{\sqrt{\frac{r^2-a^2}{\cancel{r^2}}}} = \frac{-1}{\sqrt{r^2-a^2}}$$

$$= \lim_{a\to r} \frac{-r^2-(r^2-a^2)+a^2}{-(r^2-a^2)-(r-a)a} =$$

$$= \lim_{a\to r} \frac{-r^2-r^2+a^2+a^2}{-r^2+a^2-ra+a^2} = \lim_{a\to r} \frac{2a^2-2r^2}{-r^2-ra+2a^2} =$$

$$= \left[\frac{0}{0}\right] \overset{\text{L'Hop.}}{=} \lim_{a\to r} \frac{4a}{-r+4a} = \frac{4r}{-r+4r} =$$

$$= \frac{4\cancel{r}}{3\cancel{r}} = \frac{4}{3}$$

$$\frac{A_2}{A} \xrightarrow{a\to r} \frac{4}{3}$$

$\frac{A_2}{A_1} = 3$