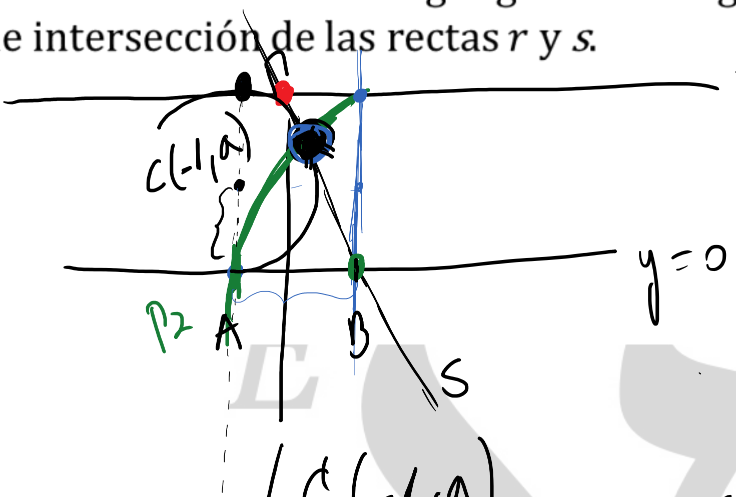
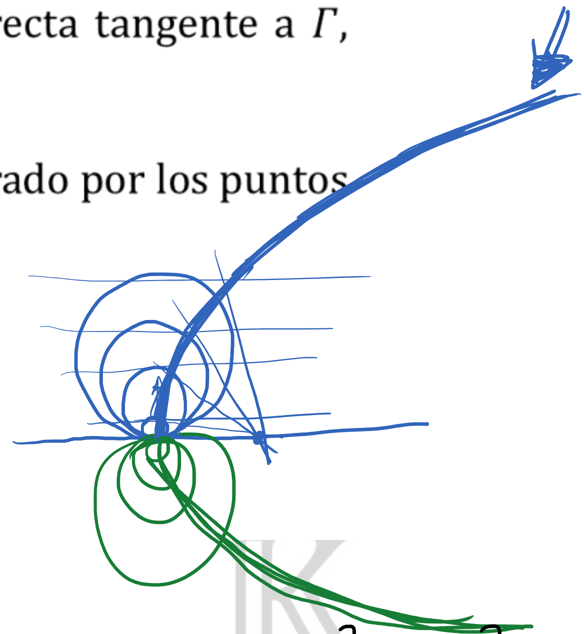
Una circunferencia variable Γ es tangente a la recta $y = 0$ en el punto $A(-1,0)$. Sea r la recta tangente a Γ en el punto diametralmente opuesto al punto A, y s la recta tangente a Γ, distinta de $y = 0$, que pasa por $B(1,0)$.

Hallar la ecuación del lugar geométrico generado por los puntos de intersección de las rectas r y s.





$$\Gamma \equiv \begin{cases} C(-1,a) \\ r=a \end{cases} \Longrightarrow (x+1)^2 + (y-a)^2 = a^2$$

$$\boxed{\Gamma_2 \equiv \begin{cases} C(1,0) \\ r=2 \end{cases} \Longrightarrow (x-1)^2 + y^2 = 2^2}$$

$$s \equiv \begin{cases} B(1,0) \\ y = mx+n \end{cases} \longrightarrow 0 = m \cdot 1 + n$$

$$\Downarrow$$

$$n = -m$$

$$y = mx - m = m(x-1)$$

$$s \equiv y = m(x-1)$$

$$\boxed{s \equiv y = m(x-1)}$$

$$s \cap \Gamma \equiv \begin{cases} y = m(x-1) \\ (x+1)^2 + (y-a)^2 = a^2 \end{cases}$$

$$(x+1)^2 + (m(x-1)-a)^2 = a^2$$

$$x^2+2x+1+m^2(x-1)^2+\cancel{a^2}-2am(x-1)=\cancel{a^2}$$

$$x^2+2x+1+m^2x^2-2m^2x+m^2-2amx+2am=0$$

$$(1+m^2)x^2+(2-2am-2m^2)x+(1+2am+m^2)=0$$

$$\Delta=0$$

$$\Delta=(2-2am-2m^2)^2-4(1+m^2)(1+2am+m^2)=$$

$$=\cancel{4}-4am-4m^2-4am+4a^2m^2+\cancel{4am^3}-4m^2+\cancel{4am^3}$$
$$\cancel{+4m^4}\cancel{-4}-8am-4m^2-4m^2\cancel{-8am^3}\cancel{-4m^4}$$

$$(-16+4a^2)m^2-16am=0$$

$$m\left[\left(-16+4a^2\right)m-16a\right]=0$$

$$\left(-16+4a^2\right)m-16a=0$$

$$m=\frac{16a}{-16+4a^2}=\frac{4a}{-4+a^2} \qquad a\neq\pm2$$

$$\boxed{s\equiv y=\frac{4a}{-4+a^2}(x-1)}$$

$$r\cap s\equiv\left\{\begin{array}{l} y=2a \\ y=\dfrac{4a}{-4+a^2}(x-1)\end{array}\right.$$

$$2a=\frac{4a}{-4+a^2}(x-1)$$

$$\frac{2\cancel{a}\left(-4+a^2\right)}{4\cancel{a}}=x-1$$

$$x=\frac{-4+a^2}{2}+1=\frac{-2+a^2}{2}$$

$$\left(\frac{-2+a^2}{2},\ 2a\right)$$

$$\begin{cases} x(a) = \dfrac{-2+a^2}{2} \\ y(a) = 2a \end{cases}$$