$\boxed{A1}$ $\quad f(x)=\dfrac{x^2+3x+4}{2x+2} \qquad x\neq -1 \qquad \text{Dom} f=\mathbb{R}-\{-1\}$

ⓐ A.V. $\displaystyle\lim_{x\to -1^-}\frac{x^2+3x+4}{2x+2}=\frac{2}{0^-}=-\infty$

$\displaystyle\lim_{x\to -1^+}\frac{x^2+3x+4}{2x+2}=\frac{2}{0^+}=+\infty$

A.V en $x=-1$

A.H. $\displaystyle\lim_{x\to\infty}\frac{x^2+3x+4}{2x+2}=\infty$ No tiene

A.O. $\displaystyle m=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{x^2+3x+4}{2x^2+2x}=\frac{1}{2}\neq 0.$

$\displaystyle n=\lim_{x\to\infty}\left[f(x)-mx\right]=\lim_{x\to\infty}\left[\frac{x^2+3x+4}{2x+2}-\frac{1}{2}x\,\frac{(x+1)}{(x+1)}\right]=$

$\displaystyle=\lim_{x\to\infty}\frac{\cancel{x^2}+3x+4-x\cdot(\cancel{x}+1)}{2x+2}=$

$\displaystyle=\lim_{x\to\infty}\frac{2x+4}{2x+2}=1$

$\Longrightarrow$ A. O. $\boxed{y=\frac{1}{2}x+1}$

ⓑ $f(x)=\dfrac{x^2+3x+4}{2x+2}$

$f'(x)=\dfrac{(2x+3)(2x+2)-(x^2+3x+4)\cdot 2}{(2x+2)^2}=$

$=\dfrac{4x^2+10x+6-2x^2-6x-8}{(2x+2)^2}=\dfrac{2x^2+4x-2}{(2x+2)^2}$

$$f'(x)=0 \Longleftrightarrow 2x^2+4x-2=0$$

$$x^2+2x-1=0 \qquad x=\frac{-2\pm\sqrt{4+4}}{2} \Rightarrow \boxed{x=-1\pm\sqrt{2}}$$

$$\begin{array}{c|ccccccc} f' & + & & - & & - & & + \\ \hline & & -1-\sqrt{2} & & -1 & & -1+\sqrt{2} & \\ & \nearrow & & \searrow & & \searrow & & \nearrow \end{array}$$

Crece $(-\infty,-1-\sqrt{2})\cup(-1+\sqrt{2},+\infty)$

Decrece $(-1-\sqrt{2},-1)\cup(-1,-1+\sqrt{2})$

$\boxed{A2}$ $$f(x)=\frac{1+e^x}{1-e^x} \qquad (1,1)$$

$$F(x)=\int\frac{1+e^x}{1-e^x}dx=\left[\begin{array}{l} t=e^x \\ x=\ln t \\ dx=\frac{1}{t}dt\end{array}\right]=$$

$$=\int\frac{1+t}{1-t}\cdot\frac{1}{t}dt=\int\frac{1+t}{t(1-t)}dt=$$

$$=\int\frac{1}{t}dt+\int\frac{2}{1-t}dt=\ln|t|-2\ln|1-t|+k$$

$$\frac{t+1}{t(1-t)}=\frac{A}{t}+\frac{B}{1-t}=\frac{A(1-t)+Bt}{t(1-t)}$$

$$t+1=A(1-t)+Bt$$

$$t=0 \rightarrow 1=A$$
$$t=1 \rightarrow 2=B$$

$t=0 \longrightarrow 1=A$
$t=1 \longrightarrow 2=B$

$= \ln e^x - 2\ln|1-e^x| + k = x - 2\ln|1-e^x| + k$

$F(1) = \cancel{1} - 2\ln|1-e| + k = \cancel{1} \rightarrow k = 2\ln|1-e|$

$\boxed{F(x) = x - 2\ln|1-e^x| + 2\ln|1-e|}$

A3 $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad a+d=1$

$|X|=1$

$|X| = \boxed{ad - bc = 1}$

$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$AX = XA$

$AX = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -c & -d \\ a & b \end{pmatrix}$

$XA = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} b & -a \\ d & -c \end{pmatrix}$

$\boxed{-c=b}$ $\quad \boxed{a+d=1} \rightarrow 2d=1 \rightarrow \boxed{d=1/2}$

$-d=-a \rightarrow \boxed{a=d}$ $\quad \boxed{ad-bc=1}$

$b=-c$
$a=d$ $\longrightarrow d \cdot d + c \cdot c = 1$

$d^2 + c^2 = 1$

$a = d$

$d^2 + c^2 = 1$

$\boxed{a = 1/2}$

$\frac{1}{4} + c^2 = 1 \rightarrow c^2 = 3/4 \rightarrow \boxed{c = \frac{\pm\sqrt{3}}{2}}$

$b = \frac{\mp\sqrt{3}}{2}$

$$\boxed{X_1 = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \qquad X_2 = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix}}$$

$\boxed{A4}$ $r \equiv \frac{x-2}{-1} = \frac{y-2}{3} = \frac{z-1}{1}$ $\quad \pi_1 \equiv x = 0$

$\pi_2 \equiv y = 0$

a) $r \equiv \begin{cases} x = 2 - \lambda \\ y = 2 + 3\lambda \quad \lambda \in \mathbb{R} \\ z = 1 + \lambda \end{cases}$ $\quad P(2-\lambda, 2+3\lambda, 1+\lambda)$

$d(P, \pi_1) = d(P, \pi_2)$

$d(P, \pi_1) = \frac{|2-\lambda|}{\sqrt{1}}$

$d(P, \pi_2) = \frac{|2+3\lambda|}{\sqrt{1}}$

$|2-\lambda| = |2+3\lambda|$

$\cancel{2} - \lambda = \cancel{2} + 3\lambda$

0

$2 - \lambda = -2 - 3\lambda$

$\rightarrow 4\lambda = 0 \rightarrow \boxed{\lambda_1 = 0} \rightarrow \boxed{P_1(2, 2, 1)}$

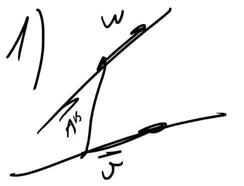
$\rightarrow 4 = -2\lambda \rightarrow \boxed{\lambda_2 = -2} \rightarrow \boxed{P_2(4, -4, -1)}$

(b)

$$r \equiv \begin{cases} x = 2 - \lambda \\ y = 2 + 3\lambda \\ z = 1 + \lambda \end{cases} \quad \lambda \in \mathbb{R} \qquad s \equiv \begin{cases} x = 0 \\ y = 0 \\ z = \alpha \end{cases} \quad \alpha \in \mathbb{R}.$$

$A(2,2,1)$ $\qquad$ $B(0,0,0)$

$\vec{v}(-1,3,1)$ $\qquad$ $\vec{w}(0,0,1)$



$\overrightarrow{BA} = (2,2,1)$

$$\begin{vmatrix} -1 & 3 & 1 \\ 0 & 0 & 1 \\ 2 & 2 & 1 \end{vmatrix} = -(-2-6) \neq 0$$

se cruzan en el espacio

B1

$f(x) = (x-a)e^x \qquad \text{Dom } f = \mathbb{R}$

(a) $f'(0) = 0 \rightarrow f'(x) = e^x + (x-a)e^x =$

$= e^x(x-a+1)$

$f'(0) = -a+1 = 0 \rightarrow \boxed{a=1}$

(b) $f(x) = (x-1)e^x \qquad f'(x) = e^x \cdot x$

$f''(x) = e^x \cdot x + e^x \cdot 1 =$

$= (x+1)e^x$

$f''(x) = 0 \rightarrow (x+1)e^x = 0 \rightarrow \cancel{e^x = 0}$

$\rightarrow x+1 = 0 \rightarrow \boxed{x=-1}$

f'': $\quad - \quad | \quad +$

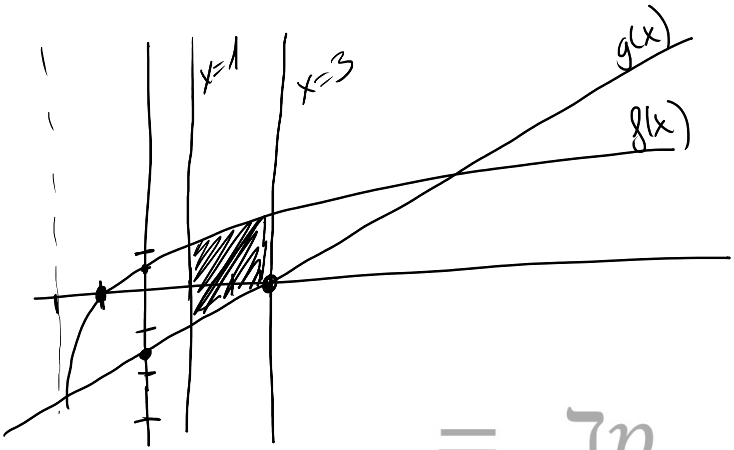
$\cap \quad -1 \quad \cup$

$\llcorner\!\!\longrightarrow x=-1$ es punto de inflexion

$x>-2$

$\boxed{B2}$ $f(x)=\ln(x+2)$

$g(x)=\frac{1}{2}(x-3)$



R.B

(b) $A=\int_1^3\left[\ln(x+2)-\frac{1}{2}(x-3)\right]dx = F(3)-F(1)=$

$=5\ln 5-0'75-3\ln 3-0'25=5\ln 5-3\ln 3-1\simeq \boxed{3'75\,u^2}$

$F(x)=\int\left[\ln(x+2)-\frac{1}{2}(x-3)\right]dx=$

$=(x+2)\ln|x+2|-x-\frac{1}{2}\left(\frac{x^2}{2}-3x\right)$

$\rightarrow F(3)=5\ln 5-0'75$

$\rightarrow F(1)=3\ln 3+0'25$

$\int\ln(x+2)\,dx=\left[\begin{array}{l} u=\ln(x+2)\rightarrow du=\frac{1}{x+2}dx \\ dv=dx \longrightarrow v=x\end{array}\right]$

$\begin{array}{r|l} x & x+2 \\ -x-2 & 1 \\ \hline -2 & \end{array}$

$=x\ln(x+2)-\int\frac{x}{x+2}dx=$

$=x\ln(x+2)-\int\left(1-\frac{2}{x+2}\right)dx=$

$$= x\ln(x+2) - x + 2\ln|x+2| =$$

$$= (x+2)\ln|x+2| - x$$

$\boxed{B3}$

$$A = \begin{pmatrix} 2-m & 1 & 2m-1 \\ 1 & m & 1 \\ m & 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 2m^2-1 \\ m \\ 1 \end{pmatrix}$$

$$X^t A = B^t$$

$$\begin{pmatrix} x & y & z \end{pmatrix}\begin{pmatrix} 2-m & 1 & 2m-1 \\ 1 & m & 1 \\ m & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2m^2-1 & m & 1 \end{pmatrix}$$

$$\begin{aligned} (2-m)x + y + mz &= 2m^2-1 \\ x + my + z &= m \\ (2m-1)x + y + z &= 1 \end{aligned}$$

$$(A^t|B^t) = \left(\begin{array}{ccc|c} 2-m & 1 & m & 2m^2-1 \\ 1 & m & 1 & m \\ 2m-1 & 1 & 1 & 1 \end{array}\right)$$

$$|A^t| = m(2-m) + m + (2m-1) - m^2(2m-1) - 1 - (2-m)$$

$$= 2m - \cancel{m^2} + m + 2m - 1 - 2m^3 \cancel{+m^2} - 1 - 2 + m$$

$$= -2m^3 + 6m - 4$$

$$|A^t| = 0 \iff -2m^3 + 6m - 4 = 0$$

$$\begin{array}{c|cccc} & -2 & 0 & 6 & -4 \\ 1 & & -2 & -2 & 4 \\ \hline & -2 & -2 & 4 & \boxed{0} \end{array}$$

$$\boxed{\begin{aligned} m &= 1 \\ m &= -2 \end{aligned}}$$

$m = 1$

$\boxed{m = -2}$

$$\begin{array}{r|rrrr} & & -2 & 2 & 1 \\ 1 & & -2 & -2 & 4 \\ \hline & & & & \boxed{0} \end{array}$$

$$\begin{array}{r|rrr} & & -2 & -4 \\ 1 & & -4 & \boxed{0} \\ \hline & -2 & & \end{array}$$

$$\begin{array}{r|rr} -2 & & 4 \\ \hline & -2 & \boxed{0} \end{array}$$

• Si $m \neq 1, -2 \rightarrow |A^t| \neq 0 \rightarrow rg(A^t) = 3$

$rg(A^t|B^t) = 3$

n^o incg $= 3$

Th. R-F

S.C.D.

• Si $m = 1 \rightarrow$ S.C.I.

$$(A^t|B^t) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} \\ \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} \end{array}\right)$$

$$\boxed{rg(A^t) = 1 = rg(A^t|B^t) < n^o \text{ inc} \rightarrow \text{S.C.I.}}$$

R-F

• Si $m = -2 \rightarrow$ S.I.

$$(A^t|B^t) = \left(\begin{array}{ccc|c} 4 & 1 & -2 & 7 \\ 1 & -2 & 1 & -2 \\ -5 & 1 & 1 & 1 \end{array}\right)$$

$$\begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} = -3 \neq 0 \rightarrow rg(A^t) = 2$$

$$\begin{vmatrix} 1 & -2 & 7 \\ -2 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = 1 - 14 + \cancel{4} - 7 - \cancel{4} + 2 \neq 0$$

$rg(A^t|B^t) = 3$

$$\boxed{rg(A^t) \neq rg(A^t|B^t) \rightarrow \text{S.I}}$$

B4 $A(1,1,0)$ $B(1,0,2)$ $C(0,2,1)$

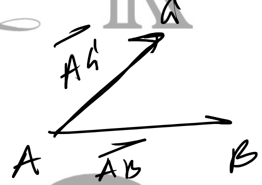
a) $$\boxed{A = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{\sqrt{14}}{2}\ u^2}$$

$$\overrightarrow{AB} = (0,-1,2) \rightarrow |\overrightarrow{AB}| = \sqrt{1+4} = \sqrt{5}$$

$$\overrightarrow{AC} = (-1,1,1) \rightarrow |\overrightarrow{AC}| = \sqrt{1+1+1} = \sqrt{3}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 0 & -1 & 2 \\ -1 & 1 & 1 \end{vmatrix} = (-3,-2,-1)$$

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{9+4+1} = \sqrt{14}$$



b) $$\cos\alpha = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AB}| \cdot |\overrightarrow{AC}|} =$$

$$= \frac{-1+2}{\sqrt{5}\cdot\sqrt{3}} = \frac{1}{\sqrt{15}} = \boxed{\frac{\sqrt{15}}{15}}$$