A1)

		A	P	€
L	x	70	20	5
E	y	60	10	4
		4200	800	

$y \geq 10$

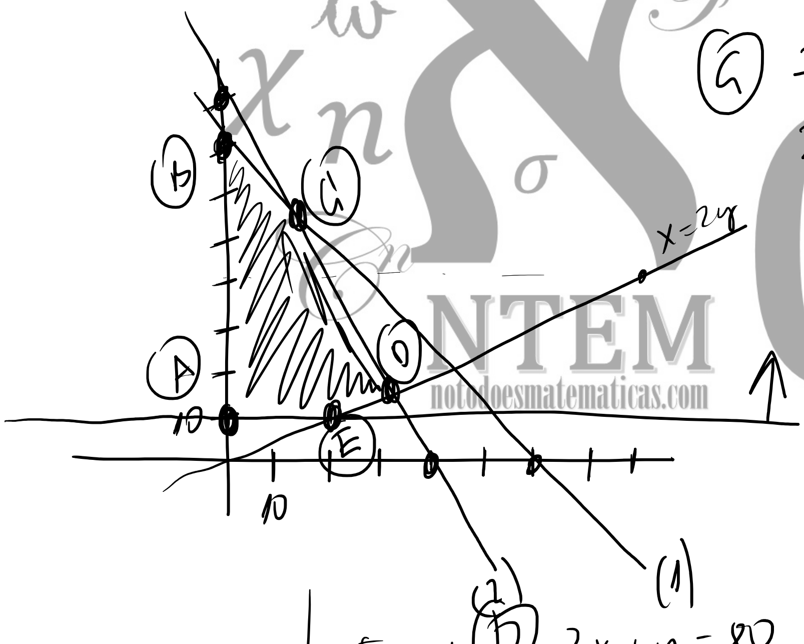
$2y \geq x$

$$\boxed{\begin{aligned} &\text{Max } f(x,y) = 5x + 4y \\ &\text{s.a. } (1)\ 70x + 60y \leq 4200 \\ &\qquad (2)\ 20x + 10y \leq 800 \\ &\qquad y \geq 10 \\ &\qquad 2y \geq x \\ &\qquad x \geq 0 \end{aligned}}$$

x	y
0	70
60	0

x	y
0	80
40	0

x	y
0	0
80	40



G $7x + 6y = 420$

$2x + y = 80$

$\boxed{y = 80 - 2x}$

$7x + 480 - 12x = 420$

$-5x = -60$

$\boxed{x = 12}$

$y = 56$

D $2x + y = 80$

$x = 2y$

$5y = 80$

$\boxed{y = 16}$

	$5x+4y$
A(0,10)	40
B(0,70)	280
G(12,56)	**284**
D(32,16)	224

Solución:

P	$(32,16)$	224
E	$(20,10)$	140

Solución:
12 lisas 56 poliéster
Beneficio 284 €

A2 $f(x)=x^3-9x+2 \qquad \parallel y=3x-3$

(a) $f'(x)=3x^2-9$

$f'(x)=3 \rightarrow 3x^2-9=3$
$3x^2=12 \rightarrow x^2=4 \rightarrow \boxed{x=\pm 2}$

$f(2)=8-18+2=-8$
$f(-2)=-8+18+2=12$

$y-f(a)=f'(a)(x-a)$

$y+8=3(x-2) \longrightarrow \boxed{y=3x-14}$
$y-12=3(x+2) \longrightarrow \boxed{y=3x+18}$

(b) $f'(x)=0 \rightarrow 3x^2-9=0 \rightarrow 3x^2=9 \rightarrow x^2=3$
$x=\pm\sqrt{3}$

f': $+$ | $-\sqrt{3}$ | $-$ | $+\sqrt{3}$ | $+$
$\nearrow$ $\searrow$ $\nearrow$

$\boxed{\begin{array}{l}\text{Crece: } (-\infty,-\sqrt{3})\cup(\sqrt{3},+\infty) \\ \text{Decrece: } (-\sqrt{3},+\sqrt{3})\end{array}}$

$f''(x)=6x \longrightarrow f''(x)=0 \Longleftrightarrow \boxed{x=0}$

f'': $-$ | $+$

$(0,+\infty)$

f'' $\quad - \quad 0 \quad +$

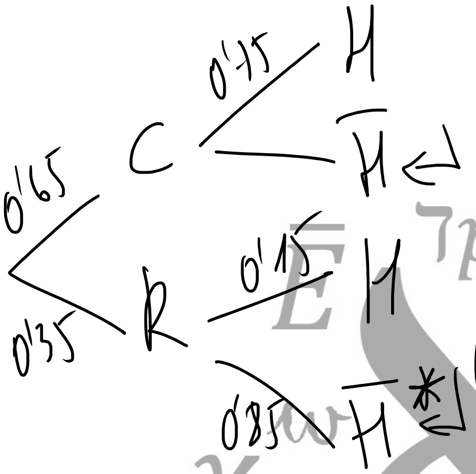
$\cap \quad 0 \quad \cup$

Concavo $(\cup)$: $(0, +\infty)$

Convexo $(\cap)$: $(-\infty, 0)$

ⓒ $\displaystyle\int (x^3 - 9x + 2)\,dx = \frac{x^4}{4} - \frac{9x^2}{2} + 2x + K.$

$\boxed{A3}$



ⓐ $p(H) = 0'65 \cdot 0'75 + 0'35 \cdot 0'15 = \boxed{= 0'54}$

ⓑ $p(R \mid \overline{H}) = \dfrac{0'35 \cdot 0'85}{1 - 0'54} = \boxed{= 0'6467}$

$\boxed{A4}$

$n = 300$

$\hat{p} = \dfrac{135}{300}$

$\alpha = 0'03$

$p \sim N\left(\hat{p}, \sqrt{\dfrac{\hat{p}\hat{q}}{n}}\right) = N(0'45, 0'029)$

$E = z_{\alpha/2} \cdot \sqrt{\dfrac{\hat{p}\hat{q}}{n}}$

$E = 2'17 \cdot 0'029 = 0'063$

$P(z \le z_{\alpha/2}) = 1 - \dfrac{\alpha}{2}$

$1 - \dfrac{0'03}{2} = 0'985 \rightarrow z_{\alpha/2} = 2'17$

$$IC = (\hat{p} - E, \hat{p} + E) =$$

$$= (0'45 - 0'063, 0'45 + 0'063)$$

$$\boxed{IC = (0'387, 0'513)}$$

(b) $E < 0'02$

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$$

$$0'02 = 2'17 \cdot \sqrt{\frac{0'45 \cdot 0'55}{n}}$$

$$\sqrt{n} = \frac{2'17 \cdot \sqrt{0'45 \cdot 0'55}}{0'02} = 53'98$$

$$n = 2913'6$$

$$\boxed{n = 2914}$$

$\boxed{B1}$

$$A = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & -1 \\ 0 & 1 & 1 \end{pmatrix} \qquad |A| = 2 - 4 + 1 = -1$$

(a) No simétrica $a_{23} \neq a_{32}$

(b) $A^{-1} = \dfrac{1}{|A|} (\mathrm{Adj}\, A)^t \qquad \mathrm{Adj}\, A = \begin{pmatrix} 3 & 2 & -2 \\ 2 & 1 & -1 \\ 2 & 1 & -2 \end{pmatrix}$

$$\boxed{A^{-1} = \begin{pmatrix} -3 & -2 & -2 \\ -2 & -1 & -1 \\ 2 & 1 & 2 \end{pmatrix}}$$

(c) $2XA - A^2 - 3I = 0$

$2XA = A^2 + 3I$

$2X = (A^2 + 3I) \cdot A^{-1} = A^2 \cdot \cancel{A^{-1}} + 3A^{-1} =$

$= A + 3A^{-1}$

$X = \dfrac{1}{2}(A + 3A^{-1})$

$$A + 3A^{-1} = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & -1 \\ 0 & 1 & 1 \end{pmatrix} + \begin{pmatrix} -9 & -6 & -6 \\ -6 & -3 & -3 \\ 6 & 3 & 6 \end{pmatrix} =$$

$$= \begin{pmatrix} -8 & -8 & -6 \\ -8 & -1 & -4 \\ 6 & 4 & 7 \end{pmatrix}$$

$$X = \begin{pmatrix} -4 & -4 & -3 \end{pmatrix}$$

$$\boxed{X = \begin{pmatrix} -4 & -4 & -3 \\ -4 & -1/2 & -2 \\ 3 & 2 & 7/2 \end{pmatrix}}$$

B2

$$f(x) = \begin{cases} \dfrac{1}{x-1} & x < 0 \\ x^2 + a & x \geq 0 \end{cases}$$

(a) $\boxed{x = 0}$

$f(0) = a$

$\lim_{x \to 0^-} \dfrac{1}{x-1} = -1$ $\quad \boxed{a = -1}$

$\lim_{x \to 0^+} x^2 + a = a$

$$f'(0^-) = \lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^-} \frac{\frac{1}{h-1} + \frac{1(h-1)}{h-1}}{h} =$$

$$= \lim_{h \to 0^-} \frac{\cancel{1} + h \cancel{- 1}}{\cancel{h}(h-1)} = \frac{1}{-1} = -1$$

$$f'(0^+) = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^+} \frac{h^2 \cancel{-1} \cancel{+1}}{\cancel{h}} = 0$$

$f'(0^-) \neq f'(0^+)$ No derivable.

$$f'(x) = \begin{cases} \dfrac{-1}{(x-1)^2} & x < 0 \end{cases} \quad f'(0^-) = -1, \quad f'(0^+) = 0$$

$$f'(x) = \begin{cases} \dfrac{\quad}{(x-1)^2} & \\ 2x & x>0 \end{cases} \qquad f'(0^+) = 0$$

$$\left(\frac{1}{x-1}\right)' = \frac{-1}{(x-1)^2}$$

b) $\boxed{a=-2}$

$$f'(x) = \begin{cases} \dfrac{-1}{(x-1)^2} & x<0 \\ 2x & x>0 \end{cases}$$

$x<0 \longrightarrow f'(x)=0 \rightarrow \dfrac{-1}{(x-1)^2}=0 \quad \times$

$x>0 \longrightarrow f'(x)=0 \rightarrow 2x=0 \longrightarrow \cancel{x=0}$

f': $-$ | 0 | $+$

$\searrow$ 0 $\nearrow$

$$\boxed{\begin{array}{ll} \text{Decrece} & (-\infty,0) \\ \text{Crece} & (0,+\infty) \end{array}}$$

$$f''(x) = \begin{cases} \dfrac{2}{(x-1)^3} & x<0 \\ 2 & x>0 \end{cases}$$

f'': $-$ | 0 | $+$

$\cap$ 0 $\cup$

Inflexión en $(0,-2)$

Inflexion en (0, 2)

Convexa (∩): $(-\infty, 0)$

Cóncavo (U): $(0, +\infty)$

~~No tiene punto de inflexión~~

B3

	S	$\overline{S}$	
P	0'22	0'13	0'35
$\overline{P}$	$\boxed{0'47}$	0'18	0'65
	0'69	0'31	

ⓐ $p = 0'22 + 0'13 + 0'47 = 0'82$

ⓑ $p = \dfrac{0'22}{0'69} = 0'3188$

ⓒ $p = 0'47$

B4

$X \equiv$ " $\sim N(\mu, 8)$

$n = 9$

$\bar{x} = 15$

ⓐ $\alpha = 0'08$

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \equiv N\left(\mu, \frac{8}{\sqrt{9}}\right) \equiv$$

$$\equiv N(\mu, 2'67)$$

$$P(Z \leq z_{\alpha/2}) = 1 - \frac{\alpha}{2}$$

$$E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$1 - \frac{0'08}{2} = 0'96 \rightarrow z_{\alpha/2} = 1'75$$

$$E = 1'75 \cdot 2'67 = \cancel{4'12}\ 4'6725$$

$$IC = (\bar{x} - E, \bar{x} + E)$$

$$IC = (15 - 4'67,\ 15 + 4'67)$$

$$\boxed{IC = (10'33,\ 19'67)}$$

ⓑ $\alpha = 0'045$

$E < 1'5$

$$E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$1'5 = 2'05 \cdot \frac{8}{\sqrt{n}}$$

$$P(Z \leq z_{\alpha/2}) = 1 - \frac{\alpha}{2}$$

$$1 - \frac{0'045}{2} = 0'9775 \rightarrow z_{\alpha/2} = 2'005$$

$$\sqrt{n} = \frac{2'005 \cdot 8}{1'5} = 10'69$$

$$n = 114'34$$

$$\boxed{\text{Sol: } n = 115}$$