A1

$$(A|b)=\left(\begin{array}{ccc|c} 1 & 1 & a & 1 \\ 1 & a & 1 & a \\ a & 1 & 1 & a+3 \end{array}\right)$$

$$|A| = a+a+a-a^3-1-1 = -a^3+3a-2$$

$$|A|=0 \rightarrow -a^3+3a-2=0$$

$$\Updownarrow$$

$$a=1 ,\; a=-2$$

$$\begin{array}{r|rrrr} & -1 & 0 & 3 & -2 \\ 1 & & -1 & -1 & 2 \\ \hline & -1 & -1 & 2 & \boxed{0} \\ 1 & & -1 & -2 & \\ \hline & -1 & -2 & \boxed{0} & \\ -2 & & +2 & & \\ \hline & +1 & \boxed{0} & & \end{array}$$

(a) Si $a \neq 1, -2 \rightarrow |A| \neq 0 \rightarrow$ $\left.\begin{array}{l} rg(A)=3 \\ rg(A|b)=3 \\ \text{nº incóg}=3 \end{array}\right|$ Th. R-F $\underline{SCD}$ solución única

$\boxed{a=0}$

$$(A|b)=\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 3 \end{array}\right) F_2-F_1$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & 1 & 1 & 3 \end{array}\right) F_3+F_2$$

$$\left| \begin{array}{ccc|c} 0 & 1 & 1 & 3 \end{array} \right| F_3+F_2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 2 & 2 \end{array} \right) \rightarrow \left. \begin{array}{r} x+y = 1 \\ -y+z = -1 \\ 2z = 2 \end{array} \right\}$$

$$\boxed{z=1} \quad \boxed{y=2} \quad \boxed{x=-1} \longrightarrow \text{Solución: } a=0 \quad \left. \begin{array}{l} x=-1 \\ y=2 \\ z=1 \end{array} \right\}$$

c) Si $\underline{a=1}$

$$(A|b) = \left(\begin{array}{ccc|c} \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 4 \end{array} \right) F_3-F_2$$

$$\begin{array}{c} F_2 \\ F_3 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 3 \end{array} \right) \quad \underline{SI}$$

b) Si $a=-2$

$|A|=0$

$rg(A) \leq 2$

$$(A|b) = \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 1 & -2 & 1 & -2 \\ \cancel{-2} & \cancel{1} & \cancel{1} & \cancel{1} \end{array} \right) F_2=F_1$$

$$\left| \begin{array}{cc} 1 & 1 \\ 1 & -2 \end{array} \right| = -3 \neq 0 \rightarrow rg(A)=2$$

$$\left| \begin{array}{ccc} 1 & 1 & 1 \\ 1 & -2 & -2 \\ -2 & 1 & 1 \end{array} \right| = 0 \rightarrow rg(A)=2$$

$\underline{SCI}$

$$\begin{vmatrix} 1 & -2 & -2 \\ -2 & 1 & 1 \end{vmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & -3 & 3 & -3 \end{array}\right) \xrightarrow{F_2 : 3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & -1 & 1 & -1 \end{array}\right)$$

$$\left.\begin{array}{r} x+y-2z=1 \\ -y+z=-1 \end{array}\right\} \xrightarrow{z=\lambda}$$

$$\left.\begin{array}{r} x+y=1+2\lambda \\ -y=-1-\lambda \end{array}\right\} \to \boxed{y=1+\lambda}$$

$$x+\cancel{1}+\lambda=\cancel{1}+2\lambda \to \boxed{x=\lambda}$$

$$\boxed{\text{Solución: } \left\{\begin{array}{l} x=\lambda \\ y=1+\lambda \\ z=\lambda \end{array}\right. \quad \lambda\in\mathbb{R}}$$

A2

ⓐ $$\int x^2\cos x\,dx=\begin{bmatrix} u=x^2 & du=2x\,dx \\ dv=\cos x\,dx & v=\operatorname{sen} x \end{bmatrix}=$$

$$=x^2\cdot\operatorname{sen} x-2\int \cancel{x}\,x\operatorname{sen} x\,dx=\begin{bmatrix} u=x & du=dx \\ dv=\operatorname{sen} x\,dx & v=-\cos x \end{bmatrix}=$$

$$=x^2\operatorname{sen} x-2\left[-x\cos x-\int -\cos x\,dx\right]=$$

$$= x^2 \operatorname{sen} x + 2x\cos x - 2\int \cos x\,dx =$$

$$= x^2 \operatorname{sen} x + 2x\cos x - 2\operatorname{sen} x + K.$$

(b) $x=0$, $x=\pi$

$$f(x)=x^2\cos x$$

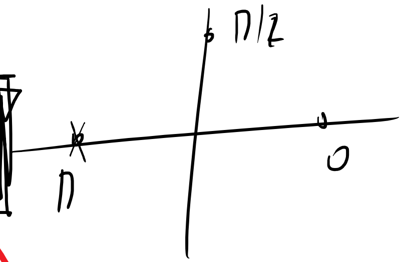
$$f(x)=0 \longrightarrow x^2\cos x=0 \begin{cases} x=0 \\ \cos x=0 \rightarrow \boxed{x=\pi/2} \end{cases}$$

R.F.

$$A = \left|\int_0^{\pi/2} f(x)\,dx\right| + \left|\int_{\pi/2}^{\pi} f(x)\,dx\right| = |F(\tfrac{\pi}{2})-F(0)|$$

$$+ |F(\pi)-F(\pi/2)| = \cancel{|\tfrac{\pi^2}{4}-0|} +$$

$$+ \cancel{|-2\pi-\tfrac{\pi^2}{4}|} = \cancel{\pi+3\pi=4\pi\ u^2}$$



$$F(x) = x^2\operatorname{sen} x + 2x\cos x - 2\operatorname{sen} x$$

$$F(0)=0$$

$$F(\tfrac{\pi}{2}) = \cancel{\pi}\ \frac{\pi^2}{4}-2$$

$$F(\pi) = -2\pi$$

$$\left|\frac{\pi^2}{4}-2\right| + \left|-2\pi-\frac{\pi^2}{4}+2\right|$$

$$\frac{\pi^2}{4}-2+\frac{\pi^2}{4}+2\pi-2 =$$

$$\frac{\pi^2}{2}+2\pi-4 \simeq$$

$$\simeq 7'22\,u^2$$

A3 $A(3,0,0)$ $B(0,3,0)$ $C(0,0,3)$

$D \in r$ $P(1,1,1)$ $\perp$ A,B,C

$$\begin{vmatrix} x-3 & y & z \\ \end{vmatrix} =$$

$$\begin{vmatrix} x-3 & y & z \end{vmatrix}$$

ⓐ $\overline{AB}=(-3,+3,0)$

$\overline{AC}=(-3,0,3)$

$A(3,0,0)$

$$\pi \equiv \begin{vmatrix} x-3 & y & z \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} =$$

$$(x-3)\cdot 1 - y(-1) + z\cdot 1 =$$

$$= x-3+y+z \Rightarrow$$

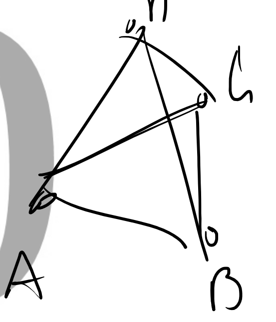
$$\boxed{\pi \equiv x+y+z-3=0}$$

ⓑ $\vec{v}(1,1,1)$

$P(1,1,1)$

$$\boxed{r \equiv \begin{cases} x = 1+\alpha \\ y = 1+\alpha \\ z = 1+\alpha \end{cases} \quad \alpha \in \mathbb{R}}$$

ⓒ $D(1+\alpha, 1+\alpha, 1+\alpha)$



$\overline{AD}(\alpha-2, 1+\alpha, 1+\alpha)$

$$V = \frac{1}{6}\left|\left[\overline{AB}, \overline{AC}, \overline{AD}\right]\right|$$

$$\left[\overline{AB}, \overline{AC}, \overline{AD}\right] = \begin{vmatrix} \alpha-2 & 1+\alpha & 1+\alpha \\ -3 & 3 & 0 \\ -3 & 0 & 3 \end{vmatrix} =$$

$$-18 \qquad +9 \qquad +9$$

$$= \overbrace{(\alpha-2)\,9} - (1+\alpha)(-9) + (1+\alpha)\,9$$

$$= 9\alpha + 9\alpha + 9\alpha = 27\alpha$$

$$V = \frac{1}{6}|27\alpha| = 18 \qquad \frac{27}{6}\alpha = 18 \rightarrow$$

$$\frac{9}{2}\alpha = 18 \rightarrow \alpha = \frac{18 \cdot 2}{9} = 4$$

$$\boxed{D(5,5,5)}$$

A4 $X \equiv$ " " $\sim N(\mu, \sigma)$

b

(1) $P(X < 5061'2) = 0'6950$

(2) $P(X > 5116'4) = 0'1660 \rightarrow P(X < 5116'4) = 0'8340$

(1) $P\left(Z < \frac{5061'2 - \mu}{\sigma}\right) = 0'6950 \rightarrow \frac{5061'2 - \mu}{\sigma} = 0'51$

(2) $P\left(Z < \frac{5116'4 - \mu}{\sigma}\right) = 0'8340 \rightarrow \frac{5116'4 - \mu}{\sigma} = 0'97$

$$5061'2 - \mu = 0'51\sigma$$

$$5116'4 - \mu = 0'97\sigma \qquad F_2 - F_1 \quad 55'2 = 0'46\sigma$$

$$\rightarrow \boxed{\sigma = 120} \quad \boxed{\mu = 5000}$$

ⓐ $P(5061'2 < X < 5116'4) =$

$P(X < 5116'4) - P(X < 5061'2) =$

$0'8340 - 0'6950 = \boxed{0'1390}$

$\boxed{B1}$ $A \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

ⓐ $A^2 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$A^4 = A^3 \cdot A = \begin{pmatrix} 1 & 3 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

ⓑ $\boxed{A^n = \begin{pmatrix} 1 & n & n \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$ $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

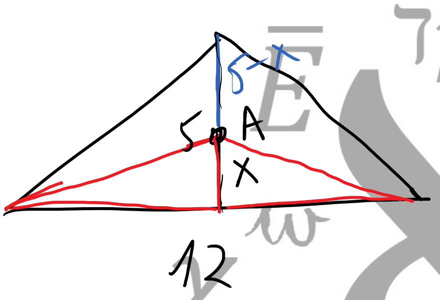
ⓒ $|A| = 1 \neq 0 \rightarrow A$ invertible

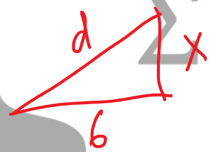
(c) $|A| = 1 \neq 0 \rightarrow A$ invertible

$$A^{-1} = \frac{1}{|A|}(\mathrm{Adj}\,A)^t \longrightarrow \quad \mathrm{Adj}\,A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$\boxed{A^{-1} = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

B2





$d^2 = x^2 + 6^2$

$d = \sqrt{x^2+36}$

(a) $f(x) = 5 - x + 2\sqrt{x^2+36}$

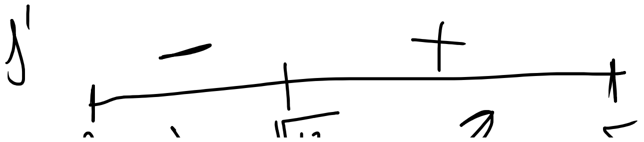
(b) $f'(x) = -1 + 2\dfrac{\cancel{2}x}{\cancel{2}\sqrt{x^2+36}}$

$f'(x) = 0 \Longleftrightarrow -1 + \dfrac{2x}{\sqrt{x^2+36}} = 0 \Longleftrightarrow \dfrac{2x}{\sqrt{x^2+36}} = 1$

$2x = \sqrt{x^2+36}$

$4x^2 = x^2 + 36 \longrightarrow 3x^2 = 36 \rightarrow x^2 = 12$

$\boxed{x = +\sqrt{12}}$



$0 \searrow \sqrt{12} \nearrow 5$

$\boxed{x = \sqrt{12} \ \text{mínimo}}$

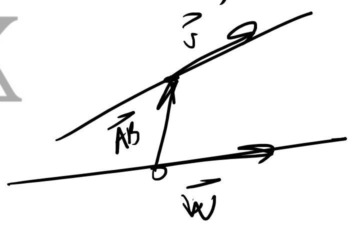
(c) $f(\sqrt{12}) = 5 - \sqrt{12} + 2\sqrt{12+36} \simeq \boxed{15'39 \ u}$

B3

$r \equiv \dfrac{x-5}{1} = \dfrac{y-6}{1} = \dfrac{z+1}{1}$ $A(5,6,-1)$, $\vec{v}(1,1,1)$

$s \equiv \dfrac{x-1}{1} = \dfrac{y}{1} = \dfrac{z+1}{-1}$ $B(1,0,-1)$, $\vec{w}(1,1,-1)$

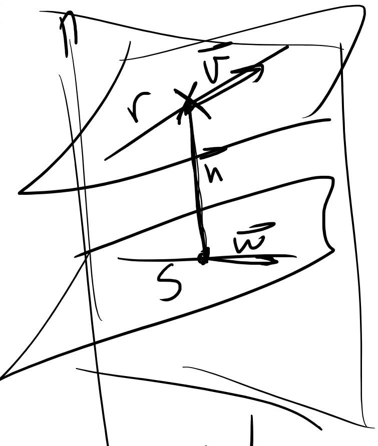
(a) $\overrightarrow{AB} = (-4,-6,0)$



$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ -4 & -6 & 0 \end{vmatrix} = -6+4+4-6 \neq 0$$

se cruzan en el espacio

(b) $\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} =$



$= (-2,2,0) \parallel (-1,1,0)$

$\pi \equiv$ $B(1,0,-1)$, $\vec{w}(1,1,-1)$, $\vec{n}(-1,1,0)$

$$\pi \equiv \begin{vmatrix} x-1 & y & z+1 \\ 1 & 1 & -1 \\ -1 & 1 & 0 \end{vmatrix} =$$

-1 $+2$

$$(x-1)(1)-y(-1)+(z+1)\cdot 2 \Rightarrow$$

$$\boxed{\pi \equiv x+y+2z+1=0}$$

$A(5,6,-1)$

$\vec{v}(1,1,1)$

$$r \equiv \begin{cases} x=5+\lambda \\ y=6+\lambda \\ z=-1+\lambda \end{cases} \quad \lambda \in \mathbb{R}$$

$$r \cap \pi \equiv 5+\lambda+6+\lambda+2(-1+\lambda)+1=0$$

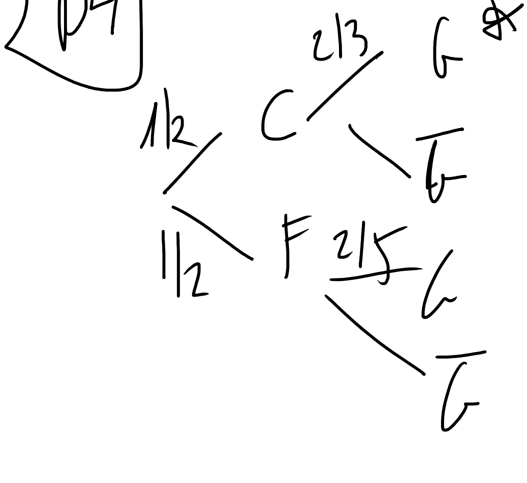
$$10+4\lambda=0 \Rightarrow \boxed{\lambda=-5/2}$$

$$r \cap \pi \left(5/2,\ 7/2,\ -7/2\right)$$

Solución: Perpendicular común:

$$\boxed{L \equiv \begin{cases} x=5/2-\alpha \\ y=7/2+\alpha \\ z=-7/2 \end{cases} \quad \alpha \in \mathbb{R}}$$

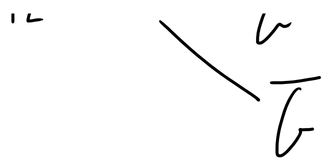
$\boxed{B4}$



a) $p(G)=\frac{1}{2}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{2}{5}=\frac{1}{3}+\frac{1}{5}=$

$$\boxed{=\frac{8}{15}\approx 0'5333}$$

b) $p(C|G)=\dfrac{\frac{1}{2}\cdot\frac{2}{3}}{8/15}=\dfrac{1/3}{8/15}=$



b) $p(a|a) = \dfrac{\frac{1}{2} \cdot \frac{2}{3}}{8/15} = \dfrac{1/3}{8/15} = \dfrac{1}{3} : \dfrac{8}{15} = \dfrac{15}{24} = \dfrac{5}{8}$

$\boxed{= 0'625}$