PROBLEMA Nº3

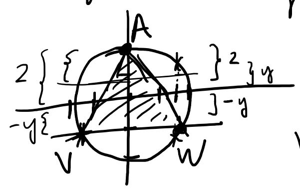
Consideremos la curva C de ecuación: $x^2 + y^2 = 4$

a) De todos los triángulos inscritos en la curva C, con vértice en el punto A(0,2) y base paralela al eje OX, calcular el que tiene máxima superficie. (5 puntos).

b) Calcular la ecuación de la envolvente de la familia de circunferencias que tienen centro en la curva C y que sus radios son la mitad del radio de C. (5 puntos).

$x^2+y^2=4 \longrightarrow C(0,0)$

$r=2$



$\cancel{W(x_1 \pm\sqrt{4-x^2})}$

$\cancel{V(-x_1 \pm\sqrt{4-x^2})}$

a) $A(0,2)$

$V, W \in x^2+y^2=4$

$V(x,y)$ $\quad \cancel{y=\pm\sqrt{4-x^2}}$

$x=\pm\sqrt{4-y^2}$

$y \in [-2,2]$

$\hookrightarrow \begin{cases} W(\sqrt{4-y^2}, y) \\ V(-\sqrt{4-y^2}, y) \end{cases}$

$A = \dfrac{b \cdot h}{2} = \cancel{\dfrac{2x(2-y)}{2}} = x(2-y)$

$$\boxed{A = \frac{\cancel{2}\sqrt{4-y^2}\cdot(2-y)}{\cancel{2}}} \Rightarrow A' = \frac{-\cancel{2}y}{\cancel{2}\sqrt{4-y^2}}(2-y) + \sqrt{4-y^2}\cdot(-1)$$

$$A' = \frac{-2y+y^2}{\sqrt{4-y^2}} - \sqrt{4-y^2} \longleftarrow$$

$$A'=0 \Longleftrightarrow \frac{-2y+y^2}{\sqrt{4-y^2}} = \sqrt{4-y^2} \Longleftrightarrow -2y+y^2 = 4-y^2 \Longleftrightarrow$$

$$2y^2-2y-4=0 \quad \begin{cases} y=2 \\ y=-1 \end{cases} \longrightarrow x=\sqrt{4-y^2}=\pm\sqrt{3}$$

A' $\quad +\quad -$

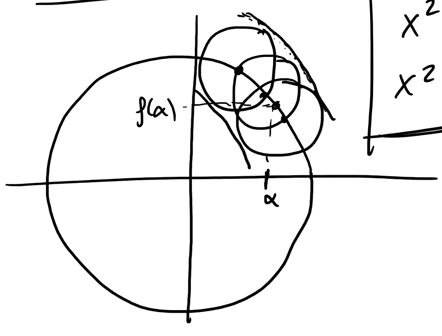
$-2 \quad -1 \quad 2$

A $\quad \nearrow \quad \searrow$

$\hookrightarrow$ Máximo para A

$$\boxed{\begin{array}{l} V(-\sqrt{3}, -1) \\ W(\sqrt{3}, -1) \\ A(0,2) \end{array}}$$

b) $x^2+y^2=4$ $\qquad r=1$



$$\boxed{\begin{array}{l} x^2+y^2=9 \\ x^2+y^2=1 \end{array}}$$

$y=\pm\sqrt{4-x^2}$

$C(\alpha, \sqrt{4-\alpha^2})$ $\qquad r=1$

$$(x-\alpha)^2 + \left(y-\sqrt{4-\alpha^2}\right)^2 = 1$$

$$F(x,y,\alpha) = (x-\alpha)^2 + \left(y-\sqrt{4-\alpha^2}\right)^2 - 1$$

$$F(x,y;\alpha) = (x-\alpha)^2 + \left(y-\sqrt{4-\alpha^2}\right)^2 - 1$$

$$\begin{cases} F = 0 \\ \dfrac{\partial F}{\partial \alpha} = 0 \end{cases} \qquad \frac{\partial F}{\partial \alpha} = -2(x-\alpha) + 2\left(y-\sqrt{4-\alpha^2}\right)\cdot \frac{+\cancel{2}\alpha}{\cancel{2}\sqrt{4-\alpha^2}}$$

$$\frac{\partial F}{\partial \alpha} = 0 \Longleftrightarrow \cancel{2}(x-\alpha) = \cancel{2}\left(y-\sqrt{4-\alpha^2}\right)\frac{\alpha}{\sqrt{4-\alpha^2}}$$

$$(x-\cancel{\alpha})\sqrt{4-\alpha^2} = y\alpha - \cancel{\alpha\sqrt{4-\alpha^2}}$$

$$\left(x\sqrt{4-\alpha^2}\right)^2 = (y\alpha)^2 \rightarrow x^2(4-\alpha^2) = y^2\alpha^2 \rightarrow$$

$$\rightarrow 4x^2 - \alpha^2x^2 = y^2\alpha^2 \rightarrow 4x^2 = \alpha^2(y^2+x^2) \rightarrow \alpha = \sqrt{\frac{4x^2}{y^2+x^2}} \Rightarrow$$

$$\boxed{\alpha = \frac{2x}{\sqrt{x^2+y^2}}} \quad \text{“}f(\alpha)\text{”} \quad \sqrt{4-\frac{4x^2}{x^2+y^2}} = \sqrt{\frac{4y^2}{x^2+y^2}} = \frac{2y}{\sqrt{x^2+y^2}}$$

$$\left(x-\frac{2x}{\sqrt{x^2+y^2}}\right)^2 + \left(y-\frac{2y}{\sqrt{x^2+y^2}}\right)^2 = 1$$

$$\left(\frac{x\sqrt{x^2+y^2}-2x}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{y\sqrt{x^2+y^2}-2y}{\sqrt{x^2+y^2}}\right)^2 = 1$$

$$\frac{x^2}{x^2+y^2}\left(\sqrt{x^2+y^2}-2\right)^2 + \frac{y^2}{x^2+y^2}\left(\sqrt{x^2+y^2}-2\right)^2 = 1$$

$$\left(\sqrt{x^2+y^2}-2\right)^2\underbrace{\left[\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}\right]}_{1} = 1 \rightarrow \left(\sqrt{x^2+y^2}-2\right)^2 = 1$$

$$\sqrt{x^2+y^2}-2 = 1 \rightarrow \sqrt{x^2+y^2} = 3 \rightarrow \boxed{x^2+y^2=9}$$

$$\sqrt{x^2+y^2}-2 = -1 \rightarrow \sqrt{x^2+y^2} = 1 \rightarrow \boxed{x^2+y^2=1}$$

$$\boxed{\text{Solución: } \begin{array}{l} x^2+y^2=9 \\ x^2+y^2=1 \end{array}}$$

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