1A)

$$f(x)=\begin{cases} ax^2+bx+2 & x\leq 1 \to \text{continua y derivable en } (-\infty,1) \\ a\sqrt{x}-\dfrac{b}{x^2} & x>1 \to \text{continua y derivable en } (1,+\infty) \end{cases}$$

- Continuidad en $x=1$

① $\exists f(1)$: $f(1)=a\cdot 1^2+b\cdot 1+2 = a+b+2$

② $\exists \lim_{x\to 1} f(x)$: $\left.\begin{array}{l} \lim_{x\to 1^-} ax^2+bx+2 = a+b+2 \\ \lim_{x\to 1^+} a\sqrt{x}-\dfrac{b}{x^2} = a-b \end{array}\right\}$

$\cancel{a}+b+2=\cancel{a}-b \to 2=-2b \to \boxed{b=-1}$

- Si $b=-1 \Rightarrow f(x)$ es continua en $\mathbb{R}$
- Si $b\neq -1 \Rightarrow f(x)$ no continua $\Rightarrow$ No derivable

$\Rightarrow$ Supongamos que $b=-1$.

$\dfrac{+1}{x^2}=+x^{-2}$

$-2x^{-3}$

$$f'(x)=\begin{cases} 2ax-1 & x<1 \\ a\dfrac{1}{2\sqrt{x}}-\dfrac{2}{x^3} & x>1 \end{cases}$$

$\left.\begin{array}{l} f'(1^-)=2a-1 \\ f'(1^+)=\dfrac{a}{2}-2 \end{array}\right| \Rightarrow f'(1^-)=f'(1^+)$

$2a-1=\dfrac{a}{2}-2$

$2a-\dfrac{a}{2}=-1$

$\dfrac{3a}{2}=-1 \to 3a=-2$

$\boxed{a=-2/3}$

Solución: f es continua y derivable en $\mathbb{R}$ si

Solución: f es continua y derivable en $\mathbb{R}$ si $a=-2/3$ y $b=-1$

(b) $f(x)=x^2-4 \qquad [-3,3].$

(1) f continua en $[-3,3]$ por ser polinomio

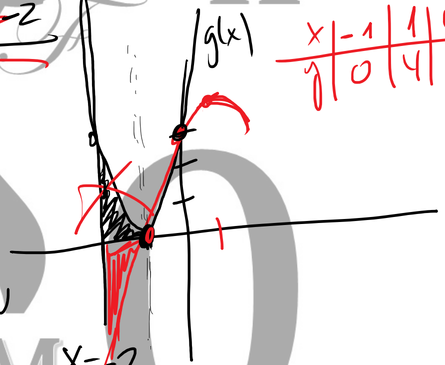
(2) f derivable en $(-3,3)$ por ser polinomio

(3) $f(-3)=(-3)^2-4=5$
$f(3)=(3)^2-4=5$ $\rightarrow f(-3)=f(3)$

SI verifica las hipótesis del Th. de Rolle.

x	-1	0	-2
y	0	3	

x	-1	1	0
y	0	4	3



2A)

a) $g(x)=-x^2+2x+3$
$x=-2$

$$A=\left|\int_{-2}^{-1} g(x)\,dx\right| \underset{\text{Regla de Barrow}}{=} \left|G(-1)-G(-2)\right| = \left|\frac{-5}{3}-\frac{2}{3}\right| = \boxed{\frac{7}{3}u^2}$$

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$$G(x)=\int(-x^2+2x+3)\,dx=\frac{-x^3}{3}+\frac{\cancel{2}x^2}{\cancel{2}}+3x$$

$$G(-1)=\frac{1}{3}+1-3=\frac{-5}{3}$$

$$G(-2)=\frac{8}{3}+4-6=\frac{8}{3}-2=\frac{2}{3}$$

b) $g(x)=-x^2+2x+3$

$$\boxed{y-g(a)=\frac{-1}{g'(a)}(x-a)}$$

$x=4$

y) $g(x) = -x^2+2x+3$

$x=4$

$g'(x) = -2x+2$

$g(4) = -16+8+3 = -5$

$g'(4) = -8+2 = -6$

$$\boxed{y-g(a) = \frac{-1}{g'(a)}(x-a)}$$

$$y-g(4) = \frac{-1}{g'(4)}(x-4)$$

$$\boxed{y+5 = \frac{-1}{-6}(x-4)}$$

$$y = \frac{1}{6}x - \frac{4}{6} - 5 = \frac{1}{6}x - \frac{34}{6} \Rightarrow \boxed{y = \frac{x-34}{6}}$$

3A)

$$(A|B) = \left(\begin{array}{ccc|c} a & 2 & 0 & a^2 \\ -1 & 1 & 1 & 5 \\ 1 & -a & -1 & -(4+a) \end{array}\right)$$

$$|A| = -a + \cancel{2} - \cancel{2} + a^2 = a^2 - a$$

$$|A| = 0 \Leftrightarrow a^2 - a = 0 \Leftrightarrow a(a-1) = 0 \rightarrow \boxed{a=0}, \ \boxed{a=1}$$

- Si $a \neq 0,1 \Rightarrow |A| \neq 0 \Rightarrow rg(A) = 3$, $rg(A|B) = 3$, nº incog $= 3$ | Th. R-F SCD

- Si $a = 0$

$|A| = 0$

$rg(A) \leq 2$

$$(A|B) = \left(\begin{array}{ccc|c} 0 & 2 & 0 & 0 \\ -1 & 1 & 1 & 5 \\ 1 & 0 & -1 & -4 \end{array}\right)$$

$$\begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} = 2 \neq 0 \longrightarrow rg(A) = 2$$

$$\begin{vmatrix} 0 & 2 & 0 \\ -1 & 1 & 5 \\ 1 & 0 & -4 \end{vmatrix} = -2 \begin{vmatrix} -1 & 5 \\ 1 & -4 \end{vmatrix} = -2 \cdot (-1) = 2 \neq 0$$

$rg(A|B) = 3$

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$$\begin{vmatrix} -1 & 1 & 5 \\ 1 & 0 & -4 \end{vmatrix} = -2 \qquad \begin{vmatrix} 1 & -4 \end{vmatrix}$$

$rg(A|B) = 3$

$\Rightarrow rg(A) = 2 \neq 3 = rg(A|B)$

por R-F $\underline{SI}$

• Si $a = 1$

$$(A|B) = \left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ -1 & 1 & 1 & 5 \\ \cancel{1} & \cancel{-1} & \cancel{-1} & \cancel{-5} \end{array}\right)$$

$\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} \neq 0 \Rightarrow rg(A) = 2$

$rg(A|B) = 2$

n^{o} inc $= 3$

$\Rightarrow$ Th. R-F $\underline{\text{E.S.C.I.}}$

b) $F_2 - F_1$ $\left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 3 & 1 & 6 \end{array}\right)$

$$\left.\begin{array}{l} x + 2y = 1 \\ 3y + z = 6 \end{array}\right\} \quad \begin{array}{l} x = 1 - 2y \\ z = 6 - 3y \\ y = y \end{array}$$

$$\boxed{\text{Solución: } \left\{\begin{array}{l} x = 1 - 2\lambda \\ y = \lambda \\ z = 6 - 3\lambda \end{array}\right. ,\ \lambda \in \mathbb{R}}$$

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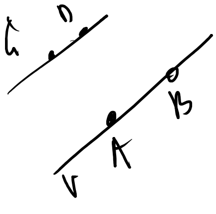
4A $A(1,2,0)$ $B(0,-1,2)$ $C(2,-1,3)$ $D(1,0,1)$



a) A $\overline{AB}$ $\overline{CD}$

$\overline{AB} = (-1, -3, 2)$

$\overrightarrow{GD} = (-1, 1, -2)$

$$\pi \equiv \begin{vmatrix} x-1 & y-2 & z \\ -1 & -3 & 2 \\ -1 & 1 & -2 \end{vmatrix} = (x-1)\,4 - (y-2)\cdot 4 + z(-4) =$$

$-4 \quad +8$

$$= 4x - 4y - 4z + 4 = 0$$

$$\boxed{\pi \equiv x - y - z + 1 = 0}$$

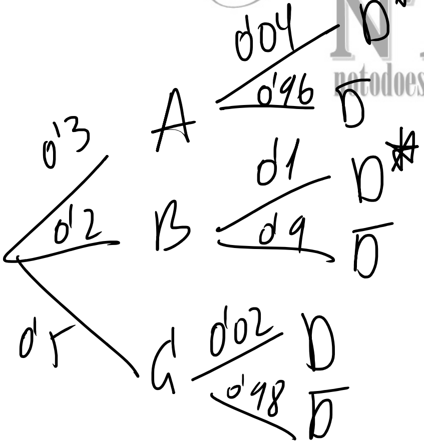
b) $\overrightarrow{AB} = (-1, -3, 2)$
$\overrightarrow{AC} = (1, -3, 3)$
$\overrightarrow{AD} = (0, -2, 1)$

$$\boxed{V_T = \frac{1}{6}\left|[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}]\right| = \cancel{\frac{10}{6} = \frac{5}{3}u^3}}$$

$$\frac{4}{6} = \frac{2}{3}u^3$$

$$\begin{vmatrix} -1 & -3 & 2 \\ 1 & -3 & 3 \\ 0 & -2 & 1 \end{vmatrix} = 3 - 4 + 3 - 6 = \cancel{-10} \quad -4$$

5A a)



a1) $P(\overline{D}) = 0'3 \cdot 0'96 + 0'2 \cdot 0'9 + 0'5 \cdot 0'98 = 0'958$

a2) $$P(\overline{G} \mid D) = \frac{P(\overline{G} \cap D)}{P(D)} = \frac{0'3 \cdot 0'04 + 0'2 \cdot 0'1}{1 - 0'958} =$$

$$= \frac{0'032}{0'042} = \frac{32}{42} = \frac{16}{21} \simeq \boxed{0'762}$$

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b) $X \equiv$ "cantidad de chicas que salen en 5 días"

$X \sim B\left(5, \frac{16}{20}\right) \equiv B(5, 0'8)$

b₁) $P(X=3) = \binom{5}{3} 0'8^3 \, 0'2^2 = 0'2048$

b₂) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= \cancel{\binom{5}{0}}^{1} 0'8^0 \, 0'2^5 + \cancel{\binom{5}{1}}^{5} 0'8^1 \, 0'2^4 +$

$+ \cancel{\binom{5}{2}}^{10} 0'8^2 \, 0'2^3 = 0'05792$

$X \equiv$ "cantidad de chicas que ... en 5 días"

$X \sim B(5, 0'2)$

b₁) $P(X=2) = 0'2048$

b₂) $P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$

$= 0'05792$

---

13) a) $\lim_{x \to 1} \left(\frac{2e^{x-1}}{x+1}\right)^{\frac{x}{x-1}} = [1^\infty]$

$= e^{\lim_{x \to 1} \frac{x}{x-1} \ln\left(\frac{2e^{x-1}}{x+1}\right)} \overset{⊛}{=} \boxed{e^{1/2}}$

⊛ $\lim_{x \to 1} \frac{x}{x-1} \ln\left(\frac{2e^{x-1}}{x+1}\right) = [\infty \cdot 0] =$

L'Hôpital

$$\lim_{x\to 1} \frac{x \cdot \ln\left(\frac{2e^{x-1}}{x+1}\right)}{x-1} = \left[\frac{0}{0}\right] \overset{\text{L'Hôpital}}{=}$$

$$\lim_{x\to 1} \frac{x\left[\ln(2e^{x-1}) - \ln(x+1)\right]}{x-1} =$$

$$\lim_{x\to 1} \frac{\left[\ln(2(e^{x-1})) - \ln(x+1)\right] + x\cdot\left[\frac{\cancel{2e^{x-1}}}{\cancel{2e^{x-1}}} - \frac{1}{x+1}\right]}{1}$$

$$= \lim_{x\to 1} \ln(2e^{x-1}) - \ln(x+1) + x - \frac{x}{x+1} =$$

$$= \cancel{\ln 2} - \cancel{\ln 2} + 1 - \frac{1}{2} = \boxed{\frac{1}{2}}$$

b) $$\lim_{x\to -1} \frac{-e^{x^2-1} - x}{x^2+4x+3} = \frac{-1+1}{0} = \left[\frac{0}{0}\right] \overset{\text{L'Hôpital}}{=}$$

$$= \lim_{x\to -1} \frac{-e^{x^2-1}(2x) - 1}{2x+4} = \frac{2-1}{2} = \boxed{\frac{1}{2}}$$

2B) $f(x) = \frac{1}{1+x^2}$ $\qquad$ $g(x) = \frac{x^2}{2}$

a) $f'(x) = \frac{-2x}{(1+x^2)^2}$ $\qquad$ $g'(x) = \frac{\cancel{2}x}{\cancel{2}} = x$

$g'(x) = 0 \Leftrightarrow \boxed{x=0}$

$f'(x) = 0 \Leftrightarrow -2x = 0$

$\boxed{x=0}$

$g''(x) = 1$

$g'(x) = \ldots$

$\boxed{x=0}$

$g''(x) = 1$

$g''(0) = 1 > 0 \to x=0$ mínimo

f': $+$ | 0 | $-$

$\nearrow \quad 0 \quad \searrow$

$\hookrightarrow x=0$ máximo

f tiene máximo relativo en $(0,1)$

g tiene un mínimo relativo en $(0,0)$

b) $f(x) = g(x)$

$$\frac{1}{1+x^2} = \frac{x^2}{2}$$

$$2 = x^2(1+x^2) \to 2 = x^2 + x^4$$

$$x^4 + x^2 - 2 = 0$$

$$\boxed{t = x^2} \to t^2 + t - 2 = 0$$

$$\boxed{t=1}, \quad \boxed{t=-2}$$

$$\cancel{x^2 = -2}$$

$$x^2 = 1$$

$$x = 1 \quad x = -1$$

$$A = \left| \int_{-1}^{1} \left(\frac{x^2}{2} - \frac{1}{1+x^2} \right) dx \right| =$$

$$= \left| F(1) - F(-1) \right| = \left| \frac{1}{6} - \frac{\pi}{4} + \frac{1}{6} - \frac{\pi}{4} \right| = \boxed{-\frac{1}{3} + \frac{\pi}{2}\ u^2}$$

$$F(x) = \int \left(\frac{x^2}{2} - \frac{1}{1+x^2} \right) dx = \frac{x^3}{6} - \operatorname{arctg} x$$

$$F(1) = \frac{1}{6} - \operatorname{arctg} 1 = \frac{1}{6} - \frac{\pi}{4}$$

$$F(-1) = \frac{-1}{6} - \operatorname{arctg}(-1) = \frac{-1}{6} + \frac{\pi}{4}$$

3B) $A \begin{pmatrix} -1 & -1 & -1 \\ -1 & 1 & 0 \\ \ldots \end{pmatrix}$ $\quad A^{-1} = \frac{1}{|A|}(\operatorname{Adj} A)^t$

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3B) $A \begin{pmatrix} -1 & 1 & 0 \\ 2 & -1 & 0 \end{pmatrix}$ $A^{-1} = \frac{1}{|A|}(Adj\,A)^t$

a)

$|A| = -1 + 2 = 1$

$$Adj\,A = \begin{pmatrix} 0 & 0 & -1 \\ +1 & 2 & -3 \\ 1 & +1 & -2 \end{pmatrix} \qquad \boxed{A^{-1} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 2 & 1 \\ -1 & -3 & -2 \end{pmatrix}}$$

b) $AX - 2B = C \rightarrow AX = C + 2B \rightarrow X = A^{-1}(C + 2B)$

$$C + 2B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 4 & 4 \\ 0 & 2 & 2 \\ 2 & -2 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 5 & 5 \\ 1 & 3 & 2 \\ 2 & -1 & 6 \end{pmatrix}$$

$$X = A^{-1}(C + 2B) = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 2 & 1 \\ -1 & -3 & -2 \end{pmatrix} \begin{pmatrix} 2 & 5 & 5 \\ 1 & 3 & 2 \\ 2 & -1 & 6 \end{pmatrix} = \boxed{\begin{pmatrix} 3 & 2 & 8 \\ 4 & 5 & 10 \\ -9 & -12 & -23 \end{pmatrix}}$$

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4B) $r \equiv \frac{x-1}{3} = \frac{y}{1} = \frac{z+1}{2}$ $P(3,1,-1)$ $\pi \equiv 2x + y - z = 0$

a) $$\boxed{d = \frac{|\vec{AP} \times \vec{v}|}{|\vec{v}|} = \frac{\sqrt{21}}{\sqrt{14}} = \sqrt{\frac{21}{14}} = \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}\,u}$$

$A(1,0,-1)$ $\vec{AP} = (2,1,0)$

$\vec{v}(3,1,2)$

$$\vec{AP} \times \vec{v} = \begin{vmatrix} i & j & k \\ 2 & 1 & 0 \\ 3 & 1 & 2 \end{vmatrix} = (2,-4,-1)$$

$|\vec{AP} \times \vec{v}| = \sqrt{4 + 16 + 1} = \sqrt{21}$

$|\vec{v}| = \sqrt{9 + 1 + 4} = \sqrt{14}$

$$|\vec{v}| = \sqrt{9+1+4} = \sqrt{14}$$

b) P, Q $\qquad$ $P(3,1,-1)$

$Q \equiv r \cap \pi_2 \qquad \pi_2 \parallel \pi \qquad P \in \pi_2 \qquad \vec{n}_\pi(2,1,-1)$

$$\pi_2 \equiv 2x+y-z+D=0$$

$$2\cdot 3+1-(-1)+D=0 \rightarrow D=-8$$

$$\boxed{\pi_2 \equiv 2x+y-z-8=0}$$

$$Q \equiv \begin{cases} 2x+y-z-8=0 \\ x=1+3\lambda \\ y=+\lambda \\ z=-1+2\lambda \end{cases}$$

$$2(1+3\lambda)+(\lambda)-(-1+2\lambda)-8=0$$

$$2+6\lambda+\lambda+1-2\lambda-8=0$$

$$5\lambda-5=0$$

$$\boxed{\lambda=1}$$

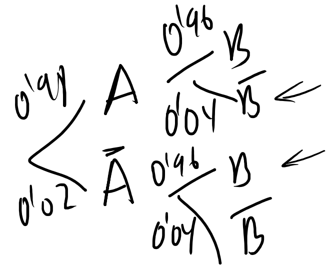
$$Q=(4,1,1)$$

$$\boxed{t \equiv \begin{cases} x=4+\alpha \\ y=1 \\ z=1+2\alpha \end{cases}, \ \alpha \in \mathbb{R}}$$

$$\overrightarrow{PQ}=(4,1,1)-(3,1,-1)=(1,0,2)$$

5B)

a)



A ≡ "actúa 1er sensor"

B ≡ "actúa 2º sensor"

a1) $P(A\cup B)=1-P(\overline{A}\cap\overline{B})=1-0'02\cdot 0'04=$

$$=0'9992$$

a2) $P(A\cap\overline{B})+P(\overline{A}\cap B)=0'98\cdot 0'04+0'02\cdot 0'96=$

$$=0'0584$$

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$a_2)$ $P(A \cap B) + P(A \cap \bar{B}) = 0'98 \cdot 0'07 + 0'02 \cdot ...$

$= \boxed{0'0584}$

b) $X \equiv$ "tiempo ..." $\sim N(10, 2)$

$b_1)$ $P(6'5 < X < 13) = P\left(\frac{6'5-10}{2} < Z < \frac{13-10}{2}\right)$

$= P(-1'75 < Z < 1'5) =$

$= P(Z < 1'5) - P(Z < -1'75) =$

$= P(Z < 1'5) - [1 - P(Z < 1'75)] =$

$= 0'9332 - 1 + 0'9599 = 0'8931$

$89'31\%$

$b_2)$ $P(X < 7) = P\left(Z < \frac{7-10}{2}\right) =$

$= P(Z < -1'5) = 1 - P(Z < 1'5) =$

$= 1 - 0'9332 = 0'0668$

$6'68\%$

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