Problema 5. *Responda razonadamente a las siguientes cuestiones:*

1. *Halle un conjunto infinito de ternas* (a, b, c) *formadas por números naturales distintos que sean solución de la ecuación*

$$a^2 + b^2 + c^2 = 2c(a + b).$$

2. *Demuestre que la ecuación*

$$a^3 + b^3 - c^3 = 3b(a - c)(a + c - b)$$

no tiene soluciones $(a, b, c) \in \mathbb{N}^3$ *tales que* $a < b < c$.

1

$$a^2+b^2+c^2=2c(a+b)$$

$$a^2+b^2+c^2=2ca+2cb$$

$$+c^2-c^2$$

$$a^2+c^2-2ca+b^2-2cb=0$$

$$(a-c)^2+(b-c)^2-c^2=0$$

$$(a-c)^2+(b-c)^2=c^2$$

$$x^2+y^2=z^2 \quad \left\{\begin{array}{l} x=a-c=3n \\ y=b-c=4n \\ z=c=5n \end{array}\right.$$

$$\boxed{c=5n}$$

$$\boxed{b=9n}$$

$$\boxed{a=8n}$$

$$(x,y,z)=(3,4,5)$$
$$=(3n,4n,5n)$$

$$\boxed{(a,b,c)=(8n,9n,5n)}$$

$$a^2+b^2+c^2=2c(a+b)$$

$$\hookrightarrow (8n)^2+(9n)^2+(5n)^2=2\cdot 5n\cdot(17n)$$

$$64n^2+81n^2+25n^2=170n^2 \quad !!$$

2

$$a^3+b^3-c^3=3b(a-c)(a+c-b) \qquad a<b<c$$

$$a^3+b^3-c^3=3b\left[(a-c)(a+c)-b(a-c)\right]=$$

$$=3b\left(a^2-c^2-ba+bc\right)=$$

$$=3ba^2-3bc^2-3b^2a+3b^2c$$

$$a^3+b^3-3ba^2+3b^2a-c^3+3bc^2-3b^2c=0 \quad -b^3+b^3$$

$$(a-b)^3+(b-c)^3+b^3=0. \qquad a<b<c$$

$$-(b-a)^3-(c-b)^3+b^3=0$$

$$\boxed{b^3=(b-a)^3+(c-b)^3}$$

$b\in\mathbb{N}$

$b-a\in\mathbb{N}$

$c-b\in\mathbb{N}$

$$x^3=y^3+z^3$$

Último Th.

$$x^n=y^n+z^n$$

$$\boxed{x^n = y^n + z^n}$$

Último Th. de Fermat

$n > 2$