ARAGÓN – 2020 – MII

$\boxed{1}$ Sea $(A|B) = \left(\begin{array}{ccc|c} 1 & 1 & m+1 & 2 \\ 1 & m-1 & 2 & 1 \\ 2 & m & 1 & -1 \end{array}\right)$ la matriz ampliada asociada al sistema.

$$|A| = m-1 + m(m+1) + 4 - 2(m-1)(m+1) - 1 - 2m =$$

$$= \cancel{m} - \cancel{1} + m^2 + \cancel{m} + 4 - 2m^2 + \cancel{2} - \cancel{1} - \cancel{2m}$$

$$= -m^2 + 4$$

$$|A| = 0 \Leftrightarrow -m^2 + 4 = 0 \Leftrightarrow \begin{array}{l} m = 2 \\ m = -2 \end{array}$$

- Si $m \neq \pm 2 \rightarrow |A| \neq 0 \rightarrow$ $\left.\begin{array}{l} rg(A) = 3 \\ rg(A|B) = 3 \\ n^o\, incóg = 3 \end{array}\right|$ $\xrightarrow{Th.\ R-F}$ $\underline{\underline{S.C.D.}}$

- Si $m = 2 \rightarrow |A| = 0$

$$(A|B) = \left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 1 & 2 & 1 \\ 2 & 2 & 1 & -1 \end{array}\right) \rightarrow \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3 \neq 0 \Downarrow rg(A) = 2$$

$$\begin{vmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -2 + 2 + 6 - 8 + 3 - 1 = 0 \rightarrow rg(A|B) = 2$$

$$rg(A) = rg(A|B) < n^o\, incóg \xrightarrow{Th.\ R-F} \underline{\underline{S.C.I.}}$$

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• Si $m=-2 \longrightarrow |A| \neq 0$

$$(A|B)=\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & -3 & 2 & 1 \\ 2 & -2 & 1 & -1 \end{array}\right) \nearrow \begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} = -4 \neq 0$$

$$\Downarrow$$

$$rg(A)=2$$

$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & -3 & 1 \\ 2 & -2 & -1 \end{vmatrix} = 4-4+2+12+2+1 \neq 0$$

$$\hookrightarrow rg(A|B)=3$$

$$rg(A) \neq rg(A|B) \xrightarrow{\text{th. R-F}} \text{S.I.}$$

$\boxed{2}$ $A\begin{pmatrix} 1 & 0 & 3 \\ -1 & 0 & 1 \end{pmatrix}$ $B\begin{pmatrix} 0 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ $C\begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix}$

(a) $$(A\cdot B^t)^{-1} = \left[\begin{pmatrix} 1 & 0 & 3 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}\right]^{-1} =$$

$$= \begin{pmatrix} 3 & 4 \\ 1 & 0 \end{pmatrix}^{-1} \quad (A\cdot B^t)^{-1} = \frac{1}{|A\cdot B^t|}\cdot (Adj\ A\cdot B^t)^t$$

$$(Adj\ A\cdot B^t) = \begin{pmatrix} 0 & -1 \\ -4 & 3 \end{pmatrix} \qquad |A\cdot B^t| = -4$$

$$(A\cdot B^t)^{-1} = \begin{pmatrix} 0 & 1 \\ 1/4 & -3/4 \end{pmatrix}$$

(b) $G^3 = I$?

$$\Rightarrow G^3 = \begin{pmatrix}-1&1\\-1&0\end{pmatrix}\begin{pmatrix}-1&1\\-1&0\end{pmatrix}\begin{pmatrix}-1&1\\-1&0\end{pmatrix} = \begin{pmatrix}0&-1\\+1&-1\end{pmatrix}\begin{pmatrix}-1&1\\-1&0\end{pmatrix} =$$

$$= \begin{pmatrix}1&0\\0&1\end{pmatrix} = I \text{ !!}$$

$$\Rightarrow G^{16} = G^{3\cdot 5+1} = (G^3)^5\cdot G = I^5\cdot G = G$$

$$\boxed{G^{16} = G}$$

$\boxed{3}$

$$X - 2Y = \begin{pmatrix}0&3&3\\0&-2&0\end{pmatrix} \quad (\cdot(-2))$$

$$2X + 3Y = \begin{pmatrix}7&6&-1\\14&3&7\end{pmatrix}$$

$$\cancel{-2X} + 4Y = \begin{pmatrix}0&-6&-6\\0&4&0\end{pmatrix}$$

$$\cancel{2X} + 3Y = \begin{pmatrix}7&6&-1\\14&3&7\end{pmatrix}$$

$$7Y = \begin{pmatrix}7&0&-7\\14&7&7\end{pmatrix} \rightarrow \boxed{Y = \begin{pmatrix}1&0&-1\\2&1&1\end{pmatrix}}$$

$$X = \begin{pmatrix}0&3&3\\0&-2&0\end{pmatrix} + 2Y = \boxed{\begin{pmatrix}2&3&1\\4&0&2\end{pmatrix}}$$

4 $r \equiv \begin{cases} x + z = 1 \\ 2x + y = 3 \end{cases} \equiv \begin{cases} z = 1 - x \\ y = 3 - 2x \\ x = x \end{cases} \equiv \begin{cases} x = \lambda \\ y = 3 - 2\lambda \\ z = 1 - \lambda \end{cases}$

a) $A(0,0,1)$

$\vec{v}_r(1,-2,-1)$

$\vec{P}_r(0,3,1)$

$\overrightarrow{AP} = (0,3,0) \parallel (0,1,0)$

$$\pi \equiv \begin{vmatrix} x & y & z-1 \\ 1 & -2 & -1 \\ 0 & 1 & 0 \end{vmatrix} = x(1) - y(0) + (z-1)1 = x + z - 1$$

$$\boxed{\pi \equiv x + z - 1 = 0}$$

5) $\vec{u}, \vec{v}$

$\vec{u} \times \vec{v}\,(-1,1,1) \rightarrow |\vec{u} \times \vec{v}| = \sqrt{3}$

$$V = |[\vec{u}, \vec{v}, \vec{u} \times \vec{v}]| = |[\vec{u} \times \vec{v}, \vec{u}, \vec{v}]| =$$

$$= |\vec{u} \times \vec{v} \cdot (\vec{u} \times \vec{v})| = |\vec{u} \times \vec{v}|^2 = 3\ u^2$$

El volumen es el producto mixto entre tres vectores.

El prod. mixto tiene la propiedad $[\vec{u}, \vec{v}, \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$.

$\boxed{5}$ $\lim\limits_{x \to 0^+} (1+x-\operatorname{sen} x)^{1/x^3} = [1^\infty] =$

$= \text{"}a^b = e^{\ln a^b} = e^{b \ln a}\text{"} =$

$= \lim\limits_{x \to 0^+} e^{\frac{1}{x^3} \ln(1+x-\operatorname{sen} x)} \overset{*}{=} e^{1/6}$

$* \lim\limits_{x \to 0^+} \frac{1}{x^3} \ln(1+x-\operatorname{sen} x) = [\infty \cdot 0] =$

$= \lim\limits_{x \to 0^+} \frac{\ln(1+x-\operatorname{sen} x)}{x^3} = \left[\frac{0}{0}\right] \overset{\text{L'Hopital}}{=}$

$= \lim\limits_{x \to 0^+} \frac{\frac{1-\cos x}{1+x-\operatorname{sen} x}}{3x^2} = \lim\limits_{x \to 0^+} \frac{1-\cos x}{3x^2(1+x-\operatorname{sen} x)} =$

$= \left[\frac{0}{0}\right] \overset{\text{L'Hopital}}{=} \lim\limits_{x \to 0^+} \frac{+\operatorname{sen} x}{6x(1+x-\operatorname{sen} x)+3x^2(1+\cos x)} =$

$= \left[\frac{0}{0}\right] \overset{\text{L'Hopital}}{=} \lim\limits_{x \to 0^+} \frac{\cos x}{6(1+x-\operatorname{sen} x)+6x(1+\cos x)+\ldots}$

$= \cancel{\lim\limits_{x \to 0}} = \frac{1}{6}$ //

6 $f(x) = \dfrac{x^2}{1-e^{-x}}$. Dom $f = \mathbb{R} - \{0\}$

• **A. vertical:** $\lim_{x \to a} f(x) = \infty \Longleftrightarrow x = a$ A.V.

$$\lim_{x \to 0} \frac{x^2}{1-e^{-x}} = \left[\frac{0}{0}\right] \overset{\text{L'Hopital}}{=} \lim_{x \to 0} \frac{2x}{e^{-x}} = \frac{0}{1} =$$

$= 0 \neq \infty \rightarrow$ **No tiene a. vertical.**

• **A. horizontal:** $\lim_{x \to \pm\infty} f(x) = k \neq \infty \Longleftrightarrow y = k$ A.H.

$$\lim_{x \to +\infty} \frac{x^2}{1-e^{-x}} = \frac{\infty}{1} = \infty$$

$$\lim_{x \to -\infty} \frac{x^2}{1-e^{-x}} = \left[\frac{\infty}{\infty}\right] \overset{\text{L'Hopital}}{=} \lim_{x \to -\infty} \frac{2x}{e^{-x}} =$$

$$= \left[\frac{\infty}{\infty}\right] \overset{\text{L'Hopital}}{=} \lim_{x \to -\infty} \frac{2}{-e^{-x}} = \frac{2}{\infty} = 0$$

$\rightarrow$ **A.H en $y = 0$** cuando $x \to -\infty$

→ A.O: $m = \lim_{x \to \pm\infty} \frac{f(x)}{x} \neq \infty$, $n = \lim_{x \to \pm\infty} [f(x) - mx] \neq \infty$ → A.O. en $y = mx + n$

→ Al ser $\lim_{x \to -\infty} f(x) = 0$ estudiamos únicamente la A.O para el caso $x \to +\infty$

→ $\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{x^2}{x(1-e^{-x})} = \frac{\infty}{1}$

$\lim_{x \to +\infty} \frac{x}{1-e^{-x}} = \frac{\infty}{1} = \infty$

No tiene a. oblicua

$\boxed{7}$ $f(x) = \ln(2x+1)$

$2x+1>0$

$x > -1/2$

(a) $\mathrm{Dom}\, f = \left(-\frac{1}{2}, +\infty\right)$

$f'(x) = \frac{2}{2x+1} \rightarrow f'(x) \neq 0 \quad \forall x \in \mathrm{Dom} f.$

f': $+$ on $\left(-1/2, +\infty\right)$, $\nearrow$

$\boxed{\text{Creciente}: \left(-\frac{1}{2}, +\infty\right)}$

(5) $y - f(\frac{1}{2}) = f'(\frac{1}{2})(x - \frac{1}{2})$

$$\left.\begin{array}{l} f(\frac{1}{2}) = \ln 2 \\ f'(\frac{1}{2}) = 1 \end{array}\right| \Rightarrow y - \ln 2 = (x - \frac{1}{2})$$

$$\boxed{y = x - \frac{1}{2} + \ln 2}$$

$\boxed{8}$ $$\int \sqrt{x}\ln^2 x\, dx = \left[\begin{array}{ll} u = \ln^2 x & du = 2\ln x \cdot \frac{1}{x}dx \\ dv = \sqrt{x}\,dx & v = \frac{x^{3/2}}{3/2} \end{array}\right]$$

$$= \frac{x^{3/2}}{3/2}\ln^2 x - \int \frac{x^{3/2}}{3/2} \cdot 2\ln x \frac{1}{x}dx =$$

$$= \frac{2x\sqrt{x}}{3}\ln^2 x - \frac{4}{3}\int x^{1/2}\ln x\, dx =$$

$$= \left[\begin{array}{ll} u = \ln x & du = \frac{1}{x}dx \\ dv = x^{1/2}dx & v = \frac{x^{3/2}}{3/2} \end{array}\right] =$$

$$= \frac{2x\sqrt{x}}{3}\ln^2 x - \frac{4}{3}\left[\frac{x^{3/2}}{3/2}\ln x - \int \frac{x^{3/2}}{3/2}\cdot\frac{1}{x}\,dx\right] =$$

$$= \frac{2x\sqrt{x}}{3}\ln^2 x - \frac{8x\sqrt{x}}{9}\ln x + \frac{8}{9}\int x^{1/2}\,dx =$$

$$= \frac{2x\sqrt{x}}{3}\ln^2 x - \frac{8x\sqrt{x}}{9}\ln x + \frac{16x\sqrt{x}}{27} + K.$$

$\boxed{9}$

$\xrightarrow{0'64} V \xrightarrow{0'12} P$

$\xrightarrow{0'36} M \xrightarrow{0'16} P$

V="varón" P="paro"

M="mujer"

ⓐ $P(M \cap P) = 0'36 \cdot 0'16 = \underline{0'0576}$

ⓑ $p(P) = p(P/M)\cdot p(M) + p(P/H)\cdot p(H) =$

$= 0'36 \cdot 0'16 + 0'64 \cdot 0'12 = \underline{\underline{0'1344}}$

ⓒ $p(M/P) = \dfrac{p(P \cap M)}{p(P)} = \dfrac{0'0576}{0'1344} = \underline{0'4286}$

$\boxed{10}$ $X =$ "# estudiantes que abandonan de entre 5"

$n = 5 \quad p = 1/5$

$X \sim B(5, 1/5)$

(a) $p(X=0) + p(X=1) = \binom{5}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^5 + \binom{5}{1}\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^4 =$

$= 0'73728$

(b) $p(X=0) = \binom{5}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^5 = 0'8^5$

$p(X=5) = \binom{5}{5}\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^0 = 0'2^5$

$0'8^5 > 0'2^5 \rightarrow$ Es más probable que ninguno abandone.