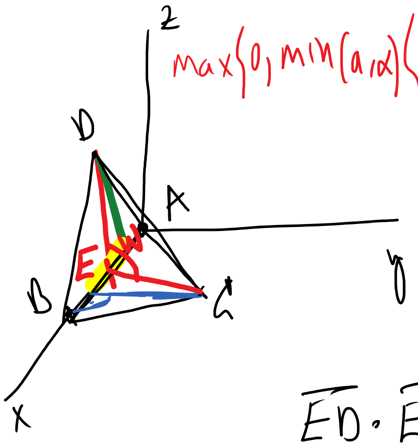
PROBLEMA 3: REGULAR

Consideramos el tetraedro de vértices A, B, C y D. Si el punto E recorre la arista AB, ¿cuándo el ángulo $C\hat{E}D$ es máximo?



$$\max\{0,\min(a,\alpha)\}\leq x\leq \min\{1,\max(a,\alpha)\}$$

$A(0,0,0)$

$B(1,0,0)$

$C(a,b,0)\quad b\neq 0$

$D(\alpha,\beta,\gamma)\quad \gamma\neq 0$

$E(x,0,0)\quad 0\leq x\leq 1$

$$\cos w=\frac{\overline{ED}\cdot\overline{EC}}{|\overline{ED}|\cdot|\overline{EC}|}$$

$$\overline{ED}=(\alpha-x,\beta,\gamma)\longrightarrow|\overline{ED}|=\sqrt{(\alpha-x)^2+\beta^2+\gamma^2}$$

$$\overline{EC}=(a-x,b,0)\longrightarrow|\overline{EC}|=\sqrt{(a-x)^2+b^2}$$

$$\overline{ED}\cdot\overline{EC}=(\alpha-x)(a-x)+\beta b$$

$$\boxed{w=\arccos\frac{(\alpha-x)(a-x)+\beta b}{\sqrt{(\alpha-x)^2+\beta^2+\gamma^2}\cdot\sqrt{(a-x)^2+b^2}}}$$

$$\begin{cases} f=(\alpha-x)(a-x)+\beta b\\ g=(\alpha-x)^2+\beta^2+\gamma^2\\ h=(a-x)^2+b^2\end{cases}$$

$\sqrt{C}$

$$w=\arccos\left(\frac{f}{\sqrt{g}\cdot\sqrt{h}}\right)$$

$$w'=\frac{-\left(f/\sqrt{g}\sqrt{h}\right)'}{\sqrt{1-\left(\frac{f}{\ldots}\right)^2}}\overset{\circledast}{=}\frac{-2f'gh+fg'h+fgh'}{2gh\sqrt{gh-f^2}}$$

$$w' = \frac{\quad}{\sqrt{1-\left(\frac{f}{\sqrt{g}\sqrt{h}}\right)^2}} = \frac{\quad}{2gh\sqrt{gh-f^2}}$$

$$\boxed{\begin{aligned}
&\left(\frac{f}{\sqrt{g}\sqrt{h}}\right)' = \frac{\left[f'\cdot\sqrt{g}\sqrt{h} - f\cdot\left[\frac{g'}{2\sqrt{g}}\cdot\sqrt{h}+\sqrt{g}\,\frac{h'}{2\sqrt{h}}\right]\right](2\sqrt{g}\sqrt{h})}{\left[gh\right](2\sqrt{g}\sqrt{h})} = \\
&= \frac{2f'gh - fg'h - fgh'}{2gh\sqrt{g}\sqrt{h}} \\
&\sqrt{1-\left(\frac{f}{\sqrt{g}\sqrt{h}}\right)^2} = \sqrt{1-\frac{f^2}{gh}} = \sqrt{\frac{gh-f^2}{gh}} = \frac{\sqrt{gh-f^2}}{\sqrt{gh}}
\end{aligned}}$$

$$w'=0 \iff \boxed{-2f'gh + fg'h + fgh' = 0}$$

$$\begin{cases}
f = (\alpha-x)(a-x)+\beta b \to f' = -(a-x)-(\alpha-x) = -a-\alpha+2x \\
g = (\alpha-x)^2+\beta^2+\gamma^2 \to g' = -2(\alpha-x) \\
h = (a-x)^2+b^2 \to h' = -2(a-x)
\end{cases}$$

$$-2f'gh + fg'h + fgh' = 0$$

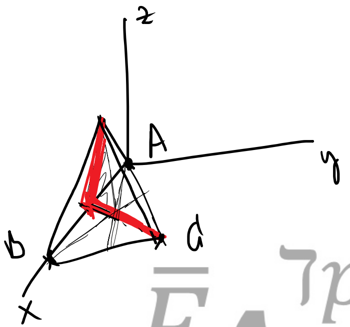
$$2f'gh = f(g'h+gh')$$

$$2\frac{f'}{f} = \frac{g'h+gh'}{gh}$$

$$2\ln|f| = \ln|gh| + C''$$

$$\ln f^2 - \ln|gh| = C'$$

$$\ln\left|\frac{f^2}{gh}\right| = C' \rightarrow \boxed{\frac{f^2}{gh} = C}$$



$A(0,0,0)$

$B(1,0,0)$

$C\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$ — a, b

$D\left(\frac{1}{2}, \frac{\sqrt{3}}{6}, \frac{\sqrt{6}}{3}\right)$ — α, β, γ

$E(x, 0, 0)$

$$W' = 0 \longleftrightarrow \boxed{-2f'gh + fg'h + fgh' = 0}$$

$$\begin{cases} f = \left(\frac{1}{2} - x\right)^2 + \frac{1}{4} \longrightarrow f' = -2\left(\frac{1}{2} - x\right) \\ g = \left(\frac{1}{2} - x\right)^2 + \frac{3}{4} \longrightarrow g' = -2\left(\frac{1}{2} - x\right) \\ h = \left(\frac{1}{2} - x\right)^2 + \frac{3}{4} \longrightarrow h' = -2\left(\frac{1}{2} - x\right) \end{cases}$$

$\{g = h$ $\qquad f' = g' = h'$ $\qquad \left| \begin{matrix} g' = f' \\ h' = f' \end{matrix} \right.$

$$-2f'h^2 + f \cdot f'h + f \cdot hf' = 0$$

$$2f'h(-h+f) = 0 \begin{cases} f' = 0 \\ h = 0 \\ h = f \end{cases}$$

$h=1$

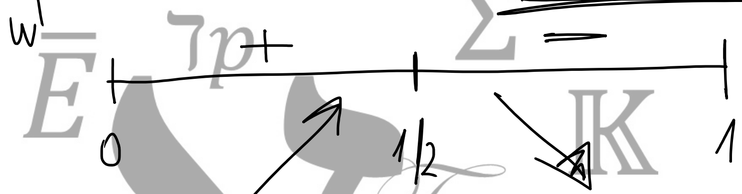
$f'=0 \Longleftrightarrow -2\left(\frac{1}{2}-x\right)=0 \Longleftrightarrow x=1/2$

$h=0 \Longleftrightarrow \left(\frac{1}{2}-x\right)^2+\frac{3}{4}=0 \Longleftrightarrow \left(\frac{1}{2}-x\right)^2=-\frac{3}{4}$ $\nexists$

$h=1 \Longleftrightarrow \cancel{\left(\frac{1}{2}-x\right)^2}+\frac{3}{4}=\cancel{\left(\frac{1}{2}-x\right)^2}+\frac{1}{4}$ $\nexists$

$\longrightarrow \boxed{x=1/2}$

$\hookrightarrow$ Máximo relativo para w



$$w'=\frac{-2f'gh+fg'h+fgh'}{2gh\sqrt{gh-f^2}}=\frac{\cancel{2}f'h(-h+f)}{\cancel{2}h^2\sqrt{h^2-f^2}}$$

$\frac{\cancel{-2}}{\cancel{-2}}\left(\frac{1}{2}-x\right)=\frac{1}{2}-x$