A1) $A\begin{pmatrix} k & 1 & 2 \\ 1 & 4 & 3 \\ 0 & 0 & 7 \end{pmatrix}$ $B\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 4 & 0 & 3 \end{pmatrix}$ $C\begin{pmatrix} 1 & 1 \\ 0 & -1 \\ 1 & 0 \end{pmatrix}$

a)
$$A-2B=\begin{pmatrix} k & 1 & 2 \\ 1 & 4 & 3 \\ 0 & 0 & 7 \end{pmatrix}-\begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 8 & 0 & 6 \end{pmatrix}=\begin{pmatrix} k-2 & 1 & 0 \\ 1 & 2 & 3 \\ -8 & 0 & 1 \end{pmatrix}$$

$$\begin{vmatrix} k-2 & 1 & 0 \\ 1 & 2 & 3 \\ -8 & 0 & 1 \end{vmatrix}=2(k-2)-24-1=$$
$$=2k-4-24-1=2k-29$$

$$2k-29=0 \Leftrightarrow \boxed{k=\frac{29}{2}}$$

b) C no invertible por no ser cuadrada

$$C^t\cdot C=\begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & -1 \\ 1 & 0 \end{pmatrix}=\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$|C^t\cdot C|=\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}=3\neq 0 \Rightarrow$$ Si es invertible

$$Adj(C^t\cdot C)=\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \qquad (C^t\cdot C)^{-1}=\begin{pmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{pmatrix}$$

$$(C^t\cdot C)^{-1}=\frac{1}{|C^t\cdot C|}\cdot\left(Adj\,(C^t\cdot C)\right)^t$$

A2)

	T	L
HE x	1	
Hora y	2	0

$x \leq$ $\boxed{15 \ l \text{ de helado}}$

$\begin{array}{c|c} x & y \\ 0 & 10 \end{array} \leftarrow x+y \geq 10 \quad 1)$

HE	x	1	
HOR	y	2	0
	10	20	

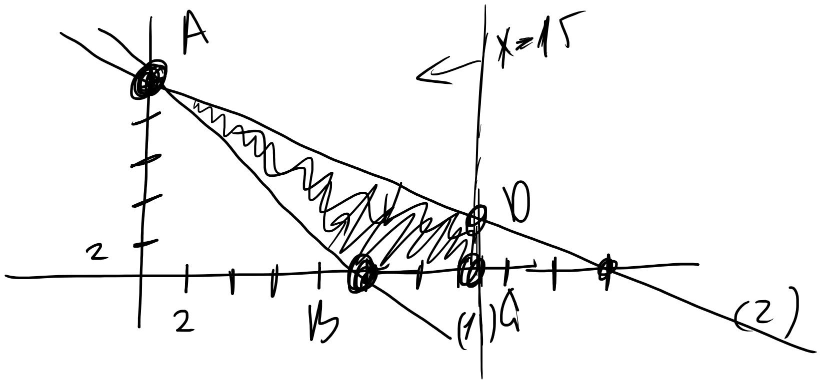
x	y
0	10
10	0

$\leftarrow x+y \geq 10 \quad (1)$

$x+2y \leq 20 \quad (2)$

x	y
0	10
20	0

(a)



(b) $25x+12y=f(x,y)$

A(0,10)	120
B(10,0)	250
C(15,0)	375
$\boxed{D(15, 2'5)}$	405

Sol: 15 l. helad
2'5 l. horchata
B = 405 €

A3 $f'(x)=2x^2-4x-6$

(a) $f(x)=\int(2x^2-4x-6)\,dx=\frac{2x^3}{3}-\frac{4x^2}{2}-6x+C$

$f(0)=3 \rightarrow f(0)=C=3$

$$\boxed{f(x)=\frac{2x^3}{3}-2x^2-6x+3}$$

(b) $f'(x)=0 \Leftrightarrow 2x^2-4x-6=0$

$x^2-2x-3=0$ ↗ $x=-1 \rightarrow f(-1)=\frac{19}{3}$

$x^2 - 2x - 3 = 0$ $\quad x = 3 \rightarrow f(3) = -15$

f'	$+$	-1	$-$	3	$+$
	$\nearrow$		$\searrow$		$\nearrow$

Máximo $\left(-1, \frac{19}{3}\right)$

Mínimo $(3, -15)$

$f''(x) = 4x - 4 \qquad f''(x) = 0 \Leftrightarrow 4x - 4 = 0$

$\boxed{x = 1}$

f''	$-$	1	$+$
	$\cap$		$\cup$

Cóncavo $(\cup)$: $(1, +\infty)$

Convexo $(\cap)$: $(-\infty, 1)$

A4

$p(A) = 0'6$

$p(B) = 0'8$

$p(A \cap \bar{B}) = 0'1$

	A	$\bar{A}$	
B	$0'5$	$0'3$	$0'8$
$\bar{B}$	$0'1$	$0'1$	$0'2$
	$0'6$	$0'4$	

a) $\boxed{p(A \mid \bar{B}) = \frac{0'1}{0'2} = 0'5}$

$p(A \cap \bar{B}) = 0'1 \qquad p(A) = 0'6,\ p(\bar{B}) = 0'2 \qquad p(A) \cdot p(\bar{B}) = 0'12$

$\boxed{p(A \cap \bar{B}) \neq p(A) \cdot p(\bar{B}) \quad \text{NO son ind}}$

b) $\boxed{p(A \cup B) = 0'5 + 0'3 + 0'1 = 0'9}$

A5 $X \equiv$ "precio …" $\sim N(\mu, 7)$

a) $n = 64 \qquad \bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \equiv N(\mu, 0'875)$

$\bar{x} = 34$

$\alpha = 0'008$

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 2'65 \cdot 0'875 =$

$= 2'32$

$P(Z \leq z_{\alpha/2}) = 1 - \frac{\alpha}{2}$

$1 - \frac{0'008}{2} = 0'996$

$\downarrow$

$z_{\alpha/2} = 2'65$

$IC = (\bar{x} - E, \ \bar{x} + E) =$

$(34 - 2'32, \ 34 + 2'32) \Rightarrow$

$$\boxed{IC = (31'68, \ 36'32)}$$

(b) n?

$E < 3$

$\alpha = 0'05$

$1 - \frac{\alpha}{2} = 0'975$

$\downarrow$

$z_{\alpha/2} = 1'96$

$E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$3 = 1'96 \cdot \frac{7}{\sqrt{n}}$

$\sqrt{n} = \frac{1'96 \cdot 7}{3} = 4'57$

$n = 20'91$

Necesitamos una muestra de $\boxed{21}$ individuos

B1

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & m & -1 \\ 1 & -1 & -m \end{pmatrix}$$

(a) $|A| = m^2 - 1 - \cancel{1} - \cancel{m} + \cancel{m} + \cancel{1} = m^2 - 1$

$|A| = 0 \Leftrightarrow m^2 - 1 = 0 \Leftrightarrow m = \pm 1$

- Si $\boxed{m = \pm 1}$, $|A| = 0 \rightarrow$ S.C.I.
  $rg(A) \leq 2$

ⓑ $\boxed{m=1}$ $A=\begin{pmatrix} -1 & 1 & 1 \\ 1 & 1 & -1 \\ \cancel{1} & \cancel{-1} & \cancel{-1} \end{pmatrix} \xrightarrow{F_2+F_1} \begin{pmatrix} -1 & 1 & 1 \\ 0 & 2 & 0 \end{pmatrix}$

$$\left.\begin{array}{l} -x+\cancel{y}+z=0 \\ 2y \qquad = 0 \end{array}\right\} \begin{array}{l} \longrightarrow x=z \\ \longrightarrow y=0 \\ z=z \end{array} \Longrightarrow \boxed{\left\{\begin{array}{l} x=\lambda \\ y=0 \\ z=\lambda \end{array}\right. ,\ \lambda\in\mathbb{R}}$$

$\boxed{B2}$ $f(x)=\dfrac{8}{x^2+4}$ $\quad$ Dom $f=\mathbb{R}$

ⓐ $f'(x)=\dfrac{-8\cdot 2x}{(x^2+4)^2}=\dfrac{-16x}{(x^2+4)^2}$

$f'(x)=0 \longleftrightarrow -16x=0 \longleftrightarrow \boxed{x=0}$

f': $+$ | 0 | $-$; $\nearrow$ $\searrow$

$\boxed{\begin{array}{l} \text{Crece: } (-\infty,0) \\ \text{Decrece: } (0,+\infty) \end{array}}$

$\Rightarrow$ **No tiene A.V. porque $D f=\mathbb{R}$**

$\Rightarrow$ A.H. $\displaystyle\lim_{x\to\infty} \frac{8}{x^2+4}=\frac{8}{\infty}=0$

$\boxed{\text{A.H. en } y=0}$

ⓑ $y-f(2)=f'(2)(x-2)$ $\qquad f'(2)=\dfrac{-32}{64}=\dfrac{-1}{2}$

$\boxed{y-1=\dfrac{-1}{2}(x-2)}$ $\qquad f(2)=\dfrac{8}{8}=1$

$$\boxed{y - 1 = -\frac{1}{2}(x-2)}$$

$$\boxed{y = -\frac{1}{2}x + 2}$$

$\boxed{B3}$

$$f(x) = \begin{cases} e^x + k & x \le 0 \\ 1 - x^2 & 0 < x \le 3 \\ \dfrac{1}{x-3} & x > 3 \end{cases}$$

a) $(-\infty, 0)$ continua por ser exponencial + cte

$(0,3)$ ~ ~ polinómica

$(3,+\infty)$ $x-3=0 \rightarrow x=3 \notin (3,+\infty) \Rightarrow$ continua.

$\boxed{x=0}$ $f(0) = 1+k$

$$\left.\begin{array}{l} \lim\limits_{x \to 0^-} e^x + k = 1+k \\ \lim\limits_{x \to 0^+} 1 - x^2 = 1 \end{array}\right| \begin{array}{l} 1+k = 1 \\ \boxed{k=0} \end{array}$$

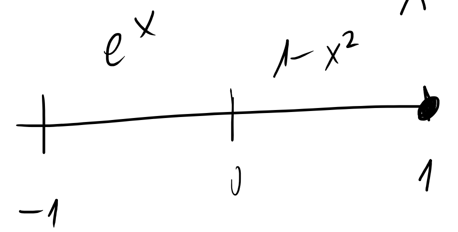
- Si $k=0 \Longrightarrow f$ continua en $x=0$
- Si $k \neq 0 \Longrightarrow f$ discontinua de salto finito en $x=0$.

$\boxed{x=3}$ $f(3) = 1 - 3^2 = -8$

$$\left.\begin{array}{l} \lim\limits_{x \to 3^-} 1 - x^2 = -8 \\ \lim\limits_{x \to 3^+} \dfrac{1}{x-3} = \dfrac{1}{0} = \infty \end{array}\right\} \text{Discontinua de salto infinito}$$

$1 \quad e^x \quad , \quad x \le 0 \longleftarrow$

ⓑ $\boxed{k=0}$ $f(x)=\begin{cases} e^x & , \quad x\le 0 \\ 1-x^2 & , \quad 0<x\le 3 \\ \dfrac{1}{x-3} & , \quad x>3 \end{cases}$



$e^x \neq 0 \rightarrow$ No se anula

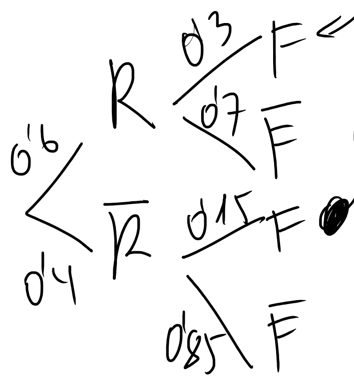
$1-x^2=0 \rightarrow x=\pm 1$

$$A=\left|\int_{-1}^{0} e^x\,dx\right|+\left|\int_0^1 (1-x^2)\,dx\right| =$$

$$=\left|\left[e^x\right]_{-1}^{0}\right|+\left|\left[x-\frac{x^3}{3}\right]_0^1\right|=\left|1-e^{-1}\right|+$$

$$+\left|1-\frac{1}{3}\right|=1-e^{-1}+\frac{2}{3}=\boxed{\frac{5}{3}-e^{-1}\ u^2}$$

$\boxed{B4}$ R = "juega" F = "fracaso"



ⓐ $P(F)=0'6\cdot 0'3+0'4\cdot 0'15=\boxed{0'24}$

ⓑ $P(\bar{R}|F)=\dfrac{P(F\cap\bar{R})}{P(F)}=\dfrac{0'4\cdot 0'15}{0'24}=$

$=\boxed{0'25}$

$\boxed{B5}$ $X\equiv$ "peso ...." $\sim N(\mu, 1'5)$

(a) $\alpha = 0'05$

$A = 0'49 \longrightarrow E = 0'245$

$\overline{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \equiv N\left(\mu, \frac{1'5}{\sqrt{n}}\right)$

$P(Z \leq z_{\alpha/2}) = 1 - \frac{\alpha}{2}$

$1 - \frac{\alpha}{2} = 0'975$

$\downarrow$

$z_{\alpha/2} = 1'96$

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$0'245 = 1'96 \cdot \frac{1'5}{\sqrt{n}}$

$\sqrt{n} = \frac{1'96 \cdot 1'5}{0'245} = 12$

$\boxed{n = 144}$

(b) $\mu = 6$

$n = 225$

$\overline{X} \sim N\left(6, \frac{1'5}{\sqrt{225}}\right) \equiv N(6, 0'1)$

$\boxed{P(\overline{X} > 5'75)} = P\left(Z > \frac{5'75 - 6}{0'1}\right) =$

$= P(Z > -2'5) = P(Z < 2'5)$

$= \boxed{0'9938}$