$\boxed{A1}$ $A = \begin{pmatrix} 1 & 3 & 4 & 1 \\ 1 & a & 2 & 2-a \\ -1 & 2 & a & a-2 \end{pmatrix}$ $M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

a) $\begin{vmatrix} 1 & 3 & 4 \\ 1 & a & 2 \\ -1 & 2 & a \end{vmatrix} = a^2+8-6+4a-3a-4 = a^2+a-2$

$a^2+a-2=0 \longleftrightarrow \begin{array}{l} a=1 \\ a=-2 \end{array}$

- Si $a \neq 1,-2 \rightarrow rg(A)=3$
- Si $a=1$ $\quad A = \begin{pmatrix} 1 & 3 & 4 & 1 \\ 1 & 1 & 2 & 1 \\ -1 & 2 & 1 & -1 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} \neq 0$ $\quad \begin{vmatrix} 1 & 3 & 1 \\ 1 & 1 & 1 \\ -1 & 2 & -1 \end{vmatrix} = 0$

$\hookrightarrow rg(A)=2$

- Si $a=-2$ $\quad A = \begin{pmatrix} 1 & 3 & 4 & 1 \\ 1 & -2 & 2 & 4 \\ -1 & 2 & -2 & -4 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 3 \\ 1 & -2 \end{vmatrix} \neq 0$ $\quad \begin{vmatrix} 1 & 3 & 1 \\ 1 & -2 & 4 \\ -1 & 2 & -4 \end{vmatrix} = 8+2-12-2+12-8=0$

$\hookrightarrow rg(A)=2$

b) $AM = \begin{pmatrix} 1 & 3 & 4 & 1 \\ 1 & 0 & 2 & 2 \\ -1 & 2 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 1 \\ 1 & 0 & 2 \\ -1 & 2 & -2 \end{pmatrix}$

$|AM| = 2-6+6-4 = -2 \neq 0 \rightarrow$ invertible

$(AM)^{-1} = \dfrac{1}{|AM|} (Adj\, AM)^t$ $\qquad Adj(AM) = \begin{pmatrix} -4 & 0 & 2 \\ +8 & -1 & -5 \\ 6 & -1 & -3 \end{pmatrix}$

$(AM)^{-1} = \begin{pmatrix} 2 & -4 & -3 \\ 0 & 1/2 & 1/2 \end{pmatrix}$

$$\boxed{(AM)^{-1} = \begin{pmatrix} 2 & -4 & -3 \\ 0 & 1/2 & 1/2 \\ -1 & 5/2 & 3/2 \end{pmatrix}}$$

A2 $\quad f(x) = \dfrac{\ln x}{x} \qquad x > 0$

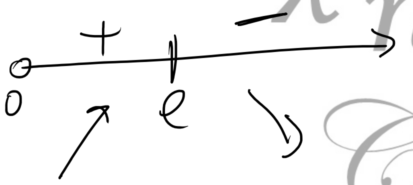
ⓐ $\displaystyle\lim_{x\to+\infty} \frac{\ln x}{x} = \left[\frac{\infty}{\infty}\right] \overset{\text{L'H.}}{=} \lim_{x\to+\infty} \frac{1/x}{1} = \lim_{x\to+\infty} \frac{1}{x} = 0.$

$\boxed{y=0 \text{ a.h. cuando } x\to+\infty.}$

ⓑ $f'(x) = \dfrac{\cancel{1/x}\cdot\cancel{x} - \ln x\cdot 1}{x^2} = \dfrac{1-\ln x}{x^2}$

$f'(x) = 0 \longleftrightarrow 1-\ln x = 0 \to \ln x = 1$

$\boxed{x = e}$



Maximo relativo en $(e, 1/e)$

ⓒ $\boxed{x=e} \qquad f(x) = 0 \longleftrightarrow \ln x = 0 \longleftrightarrow \boxed{x=1}$

$A = \left|\displaystyle\int_1^e \frac{\ln x}{x}\,dx\right| = \left|\left[\frac{\ln^2 x}{2}\right]_1^e\right| = \left|\frac{\ln^2 e}{2} - \frac{\cancel{\ln^2 1}^{\,0}}{2}\right| =$

$= \boxed{\dfrac{1}{2}\ u^2}$

A3 $\quad r \equiv \dfrac{x-1}{2} = \dfrac{y-3}{-2} = z \qquad s \equiv \begin{cases} x = 2-\alpha \\ y = -5 \\ z = 1-\alpha \end{cases},\ \alpha\in\mathbb{R}$

ⓐ ⓡ $A(1,3,0)$ $\qquad$ ⓢ $B(2,-5,1)$

$\vec{v}(2,-2,1) \qquad \vec{w}(-1,0,-1)$

(a) r: A(1,3,0) s: B(2,-5,1)
$\vec{v}(2,-2,1)$ $\vec{w}(-1,0,-1)$

$\vec{AB}=(1,-8,1)$

$$\begin{vmatrix} 2 & -2 & 1 \\ -1 & 0 & -1 \\ 1 & -8 & 1 \end{vmatrix} = 8+\cancel{2}-\cancel{2}-16 \neq 0$$ se cruzan en el espacio

(b) $\pi \parallel r$ $s \in \pi$

$B(2,-5,1)$
$\vec{w}(-1,0,-1)$
$\vec{v}(2,-2,1)$

$$\pi \equiv \begin{vmatrix} x-2 & y+5 & z-1 \\ -1 & 0 & -1 \\ 2 & -2 & +1 \end{vmatrix} =$$

$$= \overbrace{(x-2)(-2)}^{4} - \overbrace{(y+5)}^{-5} 1 + \overbrace{(z-1)}^{-2} \cdot 2$$

$$\boxed{\pi \equiv -2x-y+2z-3=0}$$

(c) $\vec{n}(2,-2,1)$
$O(0,0,0)$

$$\boxed{\pi_2 \equiv 2x-2y+z=0}$$

A4 $p=0'1$

(a) $n=10$ $X \equiv$ "cantidad de peces que sobreviven"

$X \sim B(10, 0'1)$

$$P(X \geq 2) = 1-P(X<2) = 1-[P(X=0)+P(X=1)]$$

$$= 1-\binom{10}{0}\cancel{0'1^0}\,0'9^{10}-\binom{10}{1}0'1^1\cdot 0'9^9=$$

$$\boxed{=0'264}$$

$n \cdot p$ $\sqrt{n \cdot p \cdot q}$

ⓑ $n=200$ $Y \equiv B(200, 0'1) \longrightarrow N(20, 4'24)$

$$P(Y \geq 10) = P(Y > 9'5) = P\left(Z > \frac{9'5-20}{4'24}\right) =$$

$$= P(Z > -2'48) = P(Z < 2'48) =$$

$$= \boxed{0'9934}$$

B1

B x
R y
P z

$$\begin{cases} 4x + 2y + 3z = 19 \\ x + z = 4 \\ x + y = 5 \end{cases}$$

$0'6 \cdot \alpha = 3$
$\alpha = 3/0'6 =$

$$\boxed{\begin{matrix} z = 4 - x \\ y = 5 - x \end{matrix}} \longrightarrow \begin{matrix} z = 1 \\ y = 2 \end{matrix}$$

$$4x + 10 - 2x + 12 - 3x = 19$$

$$-x = -3 \longrightarrow \boxed{x = 3}$$

Bocadillo	3€
Refrescos	2€
Patatas	1€

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2 $f(x) = \sqrt{4x^2 - x^4}$

ⓐ $4x^2 - x^4 \geq 0$ $x^2(4 - x^2) = 0$ $\begin{cases} x = 0 \\ x = 2 \\ x = -2 \end{cases}$

$-$ $+$ $+$ $-$
-2 0 2

$\text{Dom } f = [-2,2]$

(b) $$f'(x) = \frac{8x-4x^3}{2\sqrt{4x^2-x^4}}$$

$$f'(x)=0 \longleftrightarrow 8x-4x^3=0 \longleftrightarrow 4x(2-x^2)=0$$

$$\boxed{x=0,\quad x=+\sqrt{2},\quad x=-\sqrt{2}}$$

$$\begin{array}{ccccccccc} & + & & - & & + & & - & \\ -2 & \nearrow & -\sqrt{2} & \searrow & 0 & \nearrow & +\sqrt{2} & \searrow & 2 \end{array}$$

Crece $(-2,-\sqrt{2}) \cup (0,\sqrt{2})$

Decrece $(-\sqrt{2},0) \cup (\sqrt{2},2)$

(c) $$\lim_{x\to 0^-} \frac{f(x)}{x} = \lim_{x\to 0^-} \frac{\sqrt{4x^2-x^4}}{x} = \left(\frac{0}{0}\right) =$$

$$= \lim_{x\to 0^-} \frac{\sqrt{x^2(4-x^2)}}{x} = \lim_{x\to 0^-} \frac{|x|\sqrt{4-x^2}}{x} =$$

$$= \lim_{x\to 0^-} -\sqrt{4-x^2} = \boxed{-2}$$

$$\lim_{x\to 0^+} \frac{f(x)}{x} = \ldots = \lim_{x\to 0^+} \frac{\cancel{x}\sqrt{4-x^2}}{\cancel{x}} = \boxed{2}$$

B3 $A(2,1,0)$ $\quad \pi \equiv 2x+3y+4z=36$

(a) $$d(A,\pi) = \frac{|4+3-36|}{\sqrt{4+9+16}} = \frac{+29}{\sqrt{29}} = \frac{29\sqrt{29}}{29} = \underline{\overline{\sqrt{29}\,u}}$$

b) $r \equiv \begin{cases} x = 2+2\lambda \\ y = 1+3\lambda \\ z = +4\lambda \end{cases} \quad \lambda \in \mathbb{R}$

$$2(2+2\lambda)+3(1+3\lambda)+4(4\lambda)=36$$

$$4+4\lambda+3+9\lambda+16\lambda=36$$

$$29\lambda = 29 \rightarrow \boxed{\lambda=1}$$

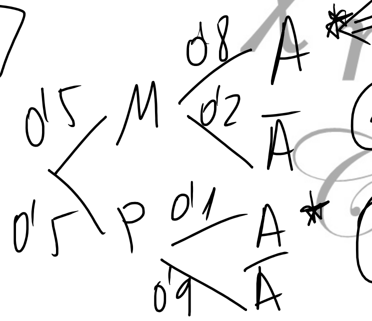
$\boxed{Q(4,4,4) \text{ proyección de } A \text{ sobre } \pi}$

c) $Q = \frac{1}{2}(A+S) \rightarrow S = 2Q - A =$

$$= (8,8,8) - (2,1,0) = (6,7,8)$$

$\boxed{\text{Solución: } (6,7,8)}$

B4



a) $p = 0'5 \cdot 0'8 + 0'5 \cdot 0'1 = \boxed{0'45}$

b) $p(M|A) = \dfrac{0'5 \cdot 0'8}{0'45} = \dfrac{0'4}{0'45} = \boxed{\approx 0'889}$