$\boxed{A1}$ ⓐ

$$A\begin{pmatrix} k & 0 & k \\ 0 & k+2 & 0 \\ 1 & 1 & k+2 \end{pmatrix}$$

$$|A| = k(k+2)^2 - k(k+2) = (k+2)\left[k^2+2k-k\right] =$$
$$= (k+2)(k^2+k) = k(k+2)(k+1)$$

$$|A| = 0 \longleftrightarrow \boxed{\begin{matrix} k=0 \\ k=-2 \\ k=-1 \end{matrix}}$$

- Si $k \neq 0, -1, -2 \Rightarrow |A| \neq 0 \rightarrow rg(A) = 3$
- Si $k=0$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix} \rightarrow \boxed{rg(A) = 2}$
- Si $k=-1$ $\begin{pmatrix} -1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ $\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -1 \neq 0$ $\boxed{rg(A) = 2}$
- Si $k=-2$ $\begin{pmatrix} -2 & 0 & -2 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$ $\begin{vmatrix} -2 & 0 \\ 1 & 1 \end{vmatrix} \neq 0$ $\boxed{rg(A) = 2}$

ⓑ $\boxed{k=1}$ $A\begin{pmatrix} 1 & 0 & 1 \\ 0 & 3 & 0 \\ \end{pmatrix}$ $|A| = 6$

b) $\boxed{k=1}$ $A\begin{pmatrix}1&0&1\\0&3&0\\1&1&3\end{pmatrix}$ $|A|=6$

$$A^{-1}=\frac{1}{|A|}\cdot(\text{Adj } A)^t \qquad \text{Adj } A=\begin{pmatrix}9&0&-3\\1&2&-1\\-3&0&3\end{pmatrix}$$

$$A^{-1}=\begin{pmatrix}\frac{9}{6}&1/6&-3/6\\0&2/6&0\\-3/6&-1/6&3/6\end{pmatrix}=\boxed{\begin{pmatrix}3/2&1/6&-1/2\\0&1/3&0\\-1/2&-1/6&1/2\end{pmatrix}}$$

A2

a) $\boxed{A(1,1,2)}$ $B(2,2,2)$ $C(-1,a,b)$

$\boxed{\vec{AB}=(1,1,0)}$

$\vec{AC}=(-2,a-1,b-2)$

$b-2=0$ $\boxed{b=2}$

$\frac{1}{-2}=\frac{1}{a-1}\longrightarrow a-1=-2$ $\boxed{a=-1}$

$$\boxed{r\equiv\begin{cases}x=1+\lambda\\y=1+\lambda\ ,\ \lambda\in\mathbb{R}\\z=2\end{cases}}$$

b) $(\vec{u}-\vec{v})\times(\vec{u}-\vec{v})=(0,0,0)$

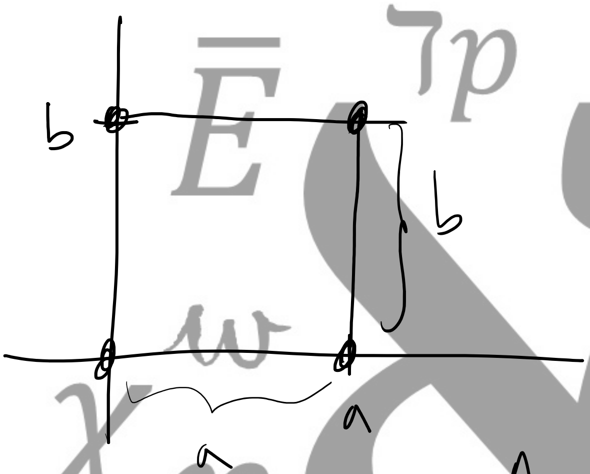
$\vec{u}-\vec{v}\ (x,y,z)$

$$\begin{vmatrix}i&j&k\\x&y&z\\x&y&z\end{vmatrix}=(0,0,0)$$

$\vec{u} - \vec{v} \quad (x, y, z)$

$$\begin{vmatrix} i & j & k \\ x & y & z \\ x & y & z \end{vmatrix} = (0, 0, 0)$$

$\boxed{A3}$ $(0,0)$ $(a,0)$ $(0,b)$ (a,b)



$y = \frac{1}{x^2} + 9$

$b = \frac{1}{a^2} + 9$

$A = a \cdot b = a\left(\frac{1}{a^2} + 9\right) = \frac{a}{a^2} + 9a \Rightarrow$

$$\boxed{A = \frac{1}{a} + 9a}$$

$A' = \frac{-1}{a^2} + 9 = 0 \Leftrightarrow \frac{1}{a^2} = 9 \rightarrow a^2 = \frac{1}{9}$

$$\boxed{a = + \frac{1}{3}}$$

$0 \quad - \quad 1/3 \quad +$

$\searrow \quad \nearrow$

$\rightarrow a = 1/3$ mínimo para A

$b = 18$

$$\boxed{A = \frac{1}{3} \cdot 18 = 6\ u^2}$$

$$\boxed{A = \frac{1}{3} \cdot 18 = 6\ u^2}$$

ⓑ $$\int \frac{1}{9-x^2}\,dx \overset{(*)}{=} \int \frac{-1/6}{x-3}\,dx + \int \frac{1/6}{x+3}\,dx =$$

(*) $9-x^2=0 \Leftrightarrow x=\pm 3 \qquad x^2-9=(x-3)(x+3)$

$$\frac{-1}{x^2-9} = \frac{A}{x-3} + \frac{B}{x+3} = \frac{A(x+3)+B(x-3)}{(x-3)(x+3)}$$

$$-1 = A(x+3)+B(x-3)$$

$x=3 \rightarrow -1 = 6A \rightarrow A = -1/6$

$x=-3 \rightarrow -1 = -6B \rightarrow B = 1/6$

$$= -\frac{1}{6}\ln|x-3| + \frac{1}{6}\ln|x+3| + C =$$

$$= \boxed{\frac{1}{6}\ln\left|\frac{x+3}{x-3}\right| + C}$$

ⓒ $$\lim_{x\to 1} \frac{x^3+x^2+kx+3}{x^3-x^2-x+1} = 2$$

$5+k=0$ $\quad \boxed{k=-5}$

$$\lim_{x\to 1} \frac{x^3+x^2+kx+3}{x^3-x^2-x+1} = \frac{5+k}{0} = \left[\frac{0}{0}\right]$$

$$\lim \frac{\cancel{(x-1)}(x^2+2x-3)}{} = \left[\frac{0}{}\right] =$$

$$\lim_{x \to 1} \frac{\cancel{(x-1)}(x^2+2x-3)}{\cancel{(x-1)}(x^2-1)} = \left[\frac{0}{0}\right] =$$

$$\begin{array}{c|cccc}
 & 1 & 1 & -5 & 3 \\
1 & & 1 & 2 & 3 \\
\hline
 & 1 & 2 & -3 & \boxed{0} \\
1 & & 1 & 3 & \\
\hline
 & 1 & 3 & \boxed{0} &
\end{array}
\qquad
\begin{array}{c|cccc}
 & 1 & -1 & -1 & 1 \\
1 & & 1 & 0 & -1 \\
\hline
 & 1 & 0 & -1 & \boxed{0} \\
1 & & 1 & 1 & \\
\hline
 & 1 & 1 & \boxed{0} &
\end{array}$$

$$\lim_{x \to 1} \frac{\cancel{(x-1)}(x+3)}{\cancel{(x-1)}(x+1)} = \boxed{\frac{4}{2} = 2}$$

Solución: $\boxed{k = -5}$

$\boxed{A4}$ $A(3M, 2R)$ $B(4M, 4R)$

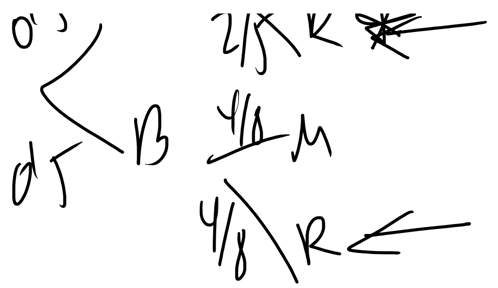
$3/5$ M — $5/9$ M, $4/9$ R

$2/5$ R — $4/9$ M, $5/9$ R

a) $p = \frac{3}{5} \cdot \frac{5}{9} + \frac{2}{5} \cdot \frac{4}{9} = \frac{15+8}{45} = \boxed{\frac{23}{45} \simeq 0'51}$

$0'5$ A — $3/5$ M, $2/5$ R

b) $P(A|R) = \frac{0'5 \cdot 2/5}{0'5 \cdot 2/5 + 0'5 \cdot \frac{4}{8}} =$



$$= \frac{0'2}{0'2+0'25} = \frac{0'2}{0'45} = \frac{4}{9} \simeq \boxed{\simeq 0'44}$$

$\boxed{B1}$

EA x
EN y
ES z

$$\left.\begin{array}{l} x+y+z=60 \\ \boxed{y+z-16=x} \\ x+z=3y \end{array}\right\}$$

$$y+z-16+z=3y$$

$$2z-16=2y \rightarrow \boxed{z-8=y}$$ *

$$z-8+z-16=x$$

$$\boxed{2z-24=x}$$ *

$\boxed{y=15}$

$\rightarrow \boxed{x=22}$

$$2z-24+z-8+z=60$$

$$4z=92 \rightarrow \boxed{z=23}$$

Solución: Alpino: 22
Nordico: 15
Escalada: 23

(b)

$$\begin{vmatrix} a & a+b & a-c \\ 2a & 3a+2b & 4a-2c \\ +3a & 6a+3b & 10a-3c \end{vmatrix} =$$

$$C_2 = \frac{b}{a} \cdot C_1 \Rightarrow 0$$

$$\begin{vmatrix} a & a & a-c \\ 2a & 3a & 4a-2c \\ +3a & 6a & 10a-3c \end{vmatrix} + \cancel{\begin{vmatrix} a & b & a-c \\ 2a & 2b & 4a-2c \\ +3a & 3b & 10a-3c \end{vmatrix}} =$$

$$C_3 = \frac{-c}{a} C_1$$

$$\begin{vmatrix} a & a & a \\ 2a & 3a & 4a \\ 3a & 6a & 10a \end{vmatrix} + \cancel{\begin{vmatrix} a & a & -c \\ 2a & 3a & -2c \\ 3a & 6a & -3c \end{vmatrix}}^{\,0} =$$

$$= a^3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix} = a^3 \left[30 + \cancel{12} + \cancel{12} - 9 - 20 - \cancel{24}\right]$$

$$= a^3 [1] = a^3$$

$$\longrightarrow \boxed{\text{Determinante} = -8}$$

B2 (a) $P(2,1,2)$ $\quad r \equiv \underbrace{(1,0,0)}_{A} + t\underbrace{(-1,1,1)}_{\vec{v}}$

$\vec{AP} = (1,1,2)$

$\vec{v} = (-1,1,1)$

$A = (1,0,0)$

$$M \equiv \begin{vmatrix} x-1 & y & z \\ 1 & 1 & 2 \\ -1 & 1 & 1 \end{vmatrix} =$$

$$(x-1)(-1)-y\cdot 3+z\cdot 2 \Rightarrow$$

$$\boxed{\Pi \equiv -x-3y+2z+1=0}$$

(b) $\vec{u}(1,2,0)$ $\vec{v}(2,1,-3)$ $\quad A=\frac{1}{2}|\vec{u}\times\vec{v}|=\boxed{\frac{3\sqrt{6}}{2}u^2}$

$$\vec{u}\times\vec{v}=\begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 2 & 1 & -3 \end{vmatrix}=(-6,3,-3)$$

$$|\vec{u}\times\vec{v}|=\sqrt{36+9+9}=\sqrt{54}=3\sqrt{6}$$

B3 $\quad f(x)=\dfrac{x-1}{(x+1)^2}$ $\quad$ Dom $f=\mathbb{R}-\{-1\}$

(a) $\displaystyle\lim_{x\to -1}\frac{x-1}{(x+1)^2}=\frac{-2}{0}=\infty \Rightarrow \boxed{A.V \text{ en } x=-1}$

$$\lim_{x\to\infty}\frac{x-1}{(x+1)^2}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{x}{x^2}=\lim_{x\to\infty}\frac{1}{x}=\frac{1}{\infty}$$

$$=0 \Rightarrow \boxed{A.H \text{ en } y=0}$$

(b) $f'(x)=\dfrac{1(x+1)^{\cancel{2}}-(x-1)\cdot 2\cancel{(x+1)}}{(x+1)^{\cancel{4}3}}=$

$$\overline{\qquad\qquad} \quad (x+1)^{\cancel{4}\,3}$$

$$= \frac{x+1-2x+2}{(x+1)^3} = \frac{-x+3}{(x+1)^3}$$

$$f'(x)=0 \longleftrightarrow -x+3=0 \longrightarrow \boxed{x=3}$$

f': $-$ | -1 | $+$ | 3 | $-$

$\searrow$ $\nearrow$ $\searrow$

Crece $(-1,3)$

Decrece $(-\infty,-1)\cup(3,+\infty)$

(c) $$\int_1^3 f(x)\,dx = \int_1^3 \frac{x-1}{(x+1)^2}\,dx = F(3)-F(1)$$

$$= \ln 4 + \frac{2}{4} - \ln 2 - \frac{2}{2} =$$

$$\boxed{= \ln 2 - \frac{1}{2}} = \ln 2 - \frac{1}{2}$$

$$F(x) = \frac{1}{2}\int \frac{2(x-1)}{(x+1)^2}\,dx = \frac{1}{2}\int \frac{2x+2-4}{(x+1)^2}\,dx = \frac{1}{2}\int \frac{2x+2}{(x+1)^2}\,dx$$

$$-\frac{4}{2}\int \frac{1}{(x+1)^2}\,dx = \ln|x+1| - 2\,\frac{(x+1)^{-1}}{-1}$$

$$\ln|x+1| + \frac{2}{x+1}$$

$$F(3) = \ln 4 + \frac{2}{4}$$

$F(1) = \ln 2 + \frac{2}{2}$

B4 $n=20$, $p=0'75$

$X \equiv$ "cantidad de mensajes SIN de entre 20"

$X \sim B(20, 0'75)$

ⓐ $P(X=10) = \binom{20}{10} 0'75^{10}\, 0'25^{10}$

ⓑ $P(X=20) = \binom{20}{20} 0'75^{20} \cdot \cancel{0'25^{0}} = 0'75^{20}$

ⓒ $P(X\leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= \binom{20}{0} \cancel{0'75^{0}}\, 0'25^{20} +$

$\binom{20}{1} 0'75^{1} \cdot 0'25^{19} +$

$\binom{20}{2} 0'75^{2} \cdot 0'25^{18} =$

$\frac{20 \cdot 19}{2} = 190$

$\frac{20!}{2!\,18!}$

$= 0'25^{20} + 20 \cdot 0'75 \cdot 0'25^{19} + 190 \cdot 0'75^{2} \cdot 0'25^{18}$