

A1)

$$A = \begin{pmatrix} k & 1 & 2 \\ 1 & 4 & 3 \\ 0 & 0 & 7 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 4 & 0 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ 1 & 0 \end{pmatrix}$$

a)

$$A - 2B = \begin{pmatrix} k & 1 & 2 \\ 1 & 4 & 3 \\ 0 & 0 & 7 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 8 & 0 & 6 \end{pmatrix} = \begin{pmatrix} k-2 & 1 & 0 \\ 1 & 2 & 3 \\ -8 & 0 & 1 \end{pmatrix}$$

$$\begin{vmatrix} k-2 & 1 & 0 \\ 1 & 2 & 3 \\ -8 & 0 & 1 \end{vmatrix} = 2(k-2) - 24 - 1 =$$

$$= 2k - 4 - 24 - 1 = 2k - 29$$

$$2k - 29 = 0 \Leftrightarrow \boxed{k = \frac{29}{2}}$$

b) C no invertible por no ser cuadrada

$$C^t \cdot C = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$|C^t \cdot C| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3 \neq 0 \Rightarrow \text{Si es invertible}$$

$$\text{Adj}(C^t \cdot C) = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \quad (C^t \cdot C)^{-1} = \begin{pmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{pmatrix}$$

$$(C^t \cdot C)^{-1} = \frac{1}{|C^t \cdot C|} \cdot (\text{Adj}(C^t \cdot C))^t$$

A2)

U E x

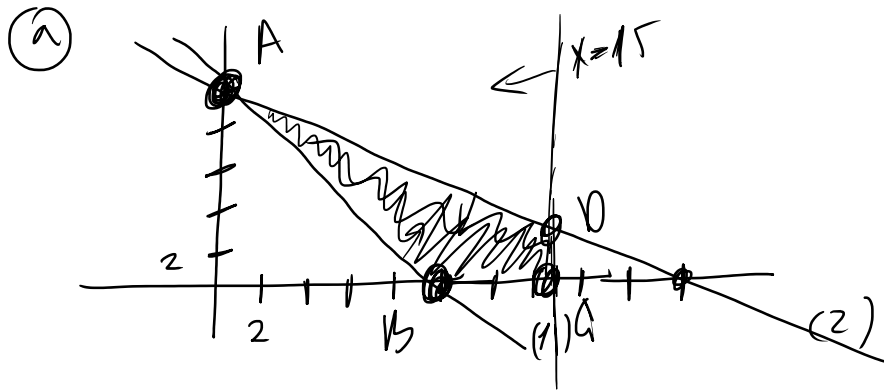
+	L
1	
0	0

$$x \leq \boxed{15 \text{ l de helado}}$$

$$\frac{x+y}{0} \geq 10 \Leftrightarrow x+y \geq 10 \quad 1)$$

$$\begin{array}{c|c|c} \text{U E } x & 1 & \\ \hline \text{HOR } y & 2 & 0 \\ \hline 10 & 20 & \end{array}$$

$$\begin{array}{c|c} x & y \\ \hline 0 & 10 \\ 10 & 0 \end{array} \leftarrow \begin{array}{l} x+y \geq 10 \quad (1) \\ x+2y \leq 20 \quad (2) \end{array}$$



(b) $25x + 12y = f(x, y)$

A(0, 10)	120
B(10, 0)	250
C(15, 0)	375
D(15, 2.5)	405

Sol: 15 l. helado
2.5 l. horcheta
B = 405 €

A3 $f'(x) = 2x^2 - 4x - 6$

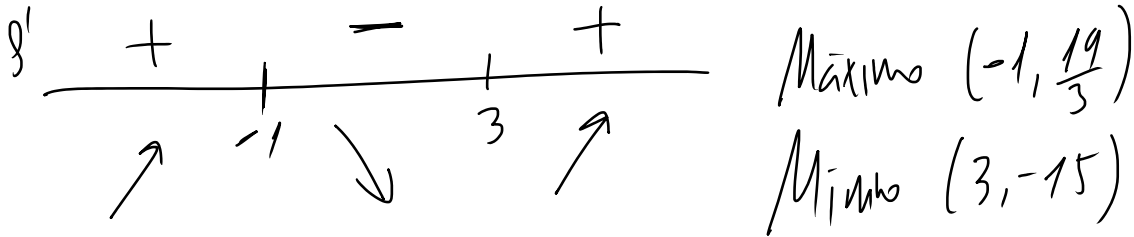
(a) $f(x) = \int (2x^2 - 4x - 6) dx = \frac{2x^3}{3} - \frac{4x^2}{2} - 6x + C$

$f(0) = 3 \rightarrow f(0) = C = 3$

$f(x) = \frac{2x^3}{3} - 2x^2 - 6x + 3$

(b) $f'(x) = 0 \Leftrightarrow 2x^2 - 4x - 6 = 0$
 $x = -1 \rightarrow f(-1) = \frac{19}{3}$

$$x = 3 \rightarrow f(3) = -15$$



$$f''(x) = 4x - 4$$

$$f''(x) = 0 \Leftrightarrow 4x - 4 = 0$$

$$\boxed{x = 1}$$



Concavo (v): $(1, +\infty)$

Concavo (^): $(-\infty, 1)$

A4

$$P(A) = 0.6$$

$$P(B) = 0.8$$

$$P(A \cap B) = 0.1$$

	A	$\bar{A}$	
B	0.5	0.3	0.8
$\bar{B}$	0.1	0.1	0.2
	0.6	0.4	

(a)
$$P(A|\bar{B}) = \frac{0.1}{0.2} = 0.5$$

$$P(A \cap \bar{B}) = 0.1$$

$$\left. \begin{array}{l} P(A) = 0.6 \\ P(\bar{B}) = 0.2 \end{array} \right\} P(A) \cdot P(\bar{B}) = 0.12$$

$$P(A \cap \bar{B}) \neq P(A) \cdot P(\bar{B}) \quad \text{NO son ind}$$

(b)
$$P(A \cup B) = 0.5 + 0.3 + 0.1 = 0.9$$

A5 $X = \text{"precio..."} \sim N(\mu, 7)$

(a) $n = 64$

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) = N(\mu, 0.875)$$

$$\bar{x} = 34$$

$$\alpha = 0.008$$

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 2.65 \cdot 0.875 = 2.32$$

$$P(z \leq z_{\alpha/2}) = 1 - \frac{\alpha}{2}$$

$$1 - \frac{0.008}{2} = 0.996$$

$$\downarrow$$

$$z_{\alpha/2} = 2.65$$

$$I\hat{G} = (\bar{x} - E, \bar{x} + E) =$$

$$(34 - 2.32, 34 + 2.32) \Rightarrow$$

$$I\hat{G} = (31.68, 36.32)$$

b) n?

$$E < 3$$

$$\alpha = 0.05$$

$$1 - \frac{\alpha}{2} = 0.975$$

$$\downarrow$$

$$z_{\alpha/2} = 1.96$$

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 = 1.96 \cdot \frac{7}{\sqrt{n}}$$

$$\sqrt{n} = \frac{1.96 \cdot 7}{3} = 4.57$$

$$n = 20.91$$

Necesitamos una muestra de 21 individuos

VB 1

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & m & -1 \\ 1 & -1 & -m \end{pmatrix}$$

$$a) |A| = m^2 - 1 - \cancel{1} - \cancel{m} + \cancel{m} + \cancel{1} = m^2 - 1$$

$$|A| = 0 \Leftrightarrow m^2 - 1 = 0 \Leftrightarrow m = \pm 1$$

• Si $m \neq \pm 1$, $|A| \neq 0 \rightarrow$ S.C.I.
 $\text{rg}(A) \leq 2$

$$m \quad \begin{vmatrix} 1 & -1 & 1 & 1 \end{vmatrix} \quad \begin{vmatrix} -1 & 1 & 1 \end{vmatrix}$$

b) $m=1$ $A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix} \xrightarrow{F_2 + F_1} \begin{pmatrix} -1 & 1 & 1 \\ 0 & 2 & 0 \\ 1 & -1 & -1 \end{pmatrix}$

$$\begin{cases} -x + y + z = 0 \\ 2y = 0 \\ z = z \end{cases} \Rightarrow \begin{cases} x = z \\ y = 0 \\ z = z \end{cases}$$

$$\begin{cases} x = \lambda \\ y = 0 \\ z = \lambda \end{cases}, \lambda \in \mathbb{R}$$

B2) $f(x) = \frac{8}{x^2+4}$ Dom $f = \mathbb{R}$

a) $f'(x) = \frac{-8 \cdot 2x}{(x^2+4)^2} = \frac{-16x}{(x^2+4)^2}$

$$f'(x) = 0 \iff -16x = 0 \iff x = 0$$

f'

Sign chart for f' :

+	0	-
↗		↘

Intervals: $(-\infty, 0)$ and $(0, +\infty)$

No tiene A.V. $Df = \mathbb{R}$

$\Rightarrow$ A.H. $\lim_{x \rightarrow \infty} \frac{8}{x^2+4} = \frac{8}{\infty} = 0$

A.H. en $y = 0$

b) $y - f(2) = f'(2)(x-2)$

$$f'(2) = \frac{-32}{64} = -\frac{1}{2}$$

$$f(2) = \frac{8}{8} = 1$$

$$y - 1 = -\frac{1}{2}(x-2)$$

$$y - 1 = -\frac{1}{2}(x - 4)$$

$$y = -\frac{1}{2}x + 2$$

$$x < 0$$

$$0 < x \leq 3$$

$$x > 3$$

B3

$$f(x) = \begin{cases} e^x + k \\ 1 - x^2 \\ \frac{1}{x-3} \end{cases}$$

- (a) $(-\infty, 0)$ Continua por ser exponencial + cte
 $(0, 3)$ ~ polinomial
 $(3, +\infty)$ $x-3=0 \rightarrow x=3 \notin (3, +\infty) \Rightarrow$ continua.

$$x=0$$

$$f(0) = 1 + k$$

$$\lim_{x \rightarrow 0^-} e^x + k = 1 + k \quad \left| \quad 1 + k = 1 \right.$$

$$\lim_{x \rightarrow 0^+} 1 - x^2 = 1 \quad \left| \quad k = 0 \right.$$

- Si $k=0 \Rightarrow f$ continua en $x=0$
- Si $k \neq 0 \Rightarrow f$ discontinua de salto finito en $x=0$.

$$x=3$$

$$f(3) = 1 - 3^2 = -8$$

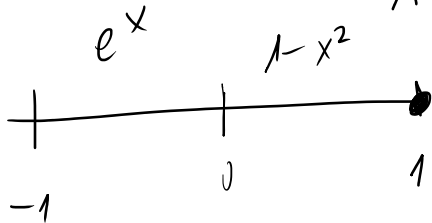
$$\lim_{x \rightarrow 3^-} 1 - x^2 = -8$$

$$\lim_{x \rightarrow 3^+} \frac{1}{x-3} = \frac{1}{0} = \infty$$

Discontinua de salto infinito

$$1 < x < 3, \quad x < 0 \leftarrow$$

⑥ k=0 $f(x) = \begin{cases} e^x & , x \leq 0 \\ 1-x^2 & , 0 < x \leq 3 \\ \frac{1}{x-3} & , x > 3 \end{cases}$



$e^x \neq 0 \rightarrow$ No se anula
 $1-x^2=0 \rightarrow x=\pm 1$

$$A = \left| \int_{-1}^0 e^x dx \right| + \left| \int_0^1 (1-x^2) dx \right| =$$

$$= \left| e^x \right|_{-1}^0 + \left| x - \frac{x^3}{3} \right|_0^1 = \left| 1 - e^{-1} \right| +$$

$$+ \left| 1 - \frac{1}{3} \right| = 1 - e^{-1} + \frac{2}{3} = \frac{5}{3} - e^{-1}$$

B4 $R \equiv$ "juega" $F \equiv$ "gana"

$\begin{matrix} 0'6 \\ 0'4 \end{matrix} \begin{matrix} R \\ \bar{R} \end{matrix}$	$\begin{matrix} 0'3 & 0'7 \\ \swarrow & \searrow \\ F & \bar{F} \end{matrix}$	$\begin{matrix} \text{a) } P(F) = 0'6 \cdot 0'3 + 0'4 \cdot 0'15 = 0'24 \\ \text{b) } P(\bar{R} F) = \frac{P(F \cap \bar{R})}{P(F)} = \frac{0'4 \cdot 0'15}{0'24} = 0'25 \end{matrix}$
	$\begin{matrix} 0'15 & 0'85 \\ \swarrow & \searrow \\ F & \bar{F} \end{matrix}$	

B5 $X \equiv$ "peso" $\sim N(\mu, 1'5)$

(a) $\alpha = 0.05$

$A = 0.49 \rightarrow E = 0.245$

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \equiv N\left(\mu, \frac{1.5}{\sqrt{n}}\right)$$

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$0.245 = 1.96 \cdot \frac{1.5}{\sqrt{n}}$$

$$\sqrt{n} = \frac{1.96 \cdot 1.5}{0.245} = 12$$

$$n = 144$$

(b) $\mu = 6$
 $n = 225$

$$\bar{X} \sim N\left(6, \frac{1.5}{\sqrt{225}}\right) \equiv N(6, 0.1)$$

$$P(\bar{X} > 5.75) = P\left(Z > \frac{5.75 - 6}{0.1}\right) =$$

$$= P(Z > -2.5) = P(Z < 2.5)$$

$$= 0.9938$$

$$P(Z \leq z_{\alpha/2}) = 1 - \frac{\alpha}{2}$$

$$1 - \frac{\alpha}{2} = 0.975$$

↓

$$z_{\alpha/2} = 1.96$$

~~AA~~