$\boxed{A1}$

$$A = \begin{pmatrix} a & 4 & 2 \\ 1 & a & 0 \\ 1 & 2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 9 \\ 1 \\ 3 \end{pmatrix}$$

a) $|A| = a^2 + \cancel{4} - 2a - \cancel{4} = a^2 - 2a = a(a-2)$

$|A| = 0 \leftrightarrow \boxed{a=0}$

$a - 2 = 0 \rightarrow \boxed{a=2}$

• Si $a = 0$ ó $2 \Rightarrow$ No invertible.

b) $\boxed{a=3}$ $\quad A^{-1}$ $\qquad AX = B$

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 1 & 3 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

$\downarrow$

$\boxed{X = A^{-1} \cdot B}$

$$A^{-1} = \frac{1}{|A|} \cdot (\text{Adj } A)^t$$

$|A| = 3$

$$\text{Adj } A = \begin{pmatrix} 3 & -1 & -1 \\ 0 & 1 & -2 \\ -6 & 2 & 5 \end{pmatrix}$$

$$\boxed{A^{-1} = \begin{pmatrix} 1 & 0 & -2 \\ -1/3 & 1/3 & 2/3 \\ -1/3 & -2/3 & 5/3 \end{pmatrix}}$$

$$\boxed{X = \begin{pmatrix} 1 & 0 & -2 \\ -1/3 & 1/3 & 2/3 \\ -1/3 & -2/3 & 5/3 \end{pmatrix} \begin{pmatrix} 9 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2/3 \\ 4/3 \end{pmatrix}}$$

$$\left.\begin{pmatrix} -1/3 & -2/3 & 5/3 \end{pmatrix} \middle| \begin{pmatrix} 3 \end{pmatrix} \quad \begin{pmatrix} 4/3 \end{pmatrix}\right]$$

A2 $f(x) = 2x^3 - 8x$

a) $f'(x) = 6x^2 - 8$ | $f'(x) = 0 \Leftrightarrow 6x^2 - 8 = 0$

$x^2 = \frac{4}{6} = \frac{2}{3}$

$x = \pm\sqrt{2/3} = \frac{\pm\sqrt{6}}{3}$

Solución: $x_1 = \frac{\sqrt{6}}{3}$

$x_2 = \frac{-\sqrt{6}}{3}$

b) $f(x) = 2x^3 - 8x \longrightarrow 2x^3 - 8x = 0$

$x(2x^2 - 8) = 0$ ⟨ $x = 0$, $x = 2$, $x = -2$

$x = 0$

$x = 2$

$A = \left|\int_0^2 (2x^3 - 8x)\,dx\right| = |F(2) - F(0)| = \boxed{8u^2}$

$F(x) = \int (2x^3 - 8x)\,dx = \frac{2x^4}{4} - \frac{8x^2}{2}$

$F(2) = 8 - 16 = -8$

$F(0) = 0$

A3 $f(x) = \begin{cases} \dfrac{x^3}{x^2 - 9} & x < 3 \end{cases}$

$x^2 - 9 = 0$

$x = \pm 3$

A) $f(x) = \begin{cases} \dfrac{x^3}{x^2-9} & \\ x^2-4 & x \geq 3 \end{cases}$ $x = \pm 3$

ⓐ $(-\infty, 3)$ discontinua en $x=-3$

$(3, +\infty)$ continua por ser polinómica

$\boxed{x=3}$ $f(3)=5$

$\lim_{x \to 3^-} \dfrac{x^3}{x^2-9} = \dfrac{27}{0} = \infty$

$\lim_{x \to 3^+} x^2-4 = 5$

Discontinua de salto infinito

en $x=-3$ ya que $\lim_{x \to -3} f(x) = \infty$

ⓑ • Vertical en $x=3$ ya que $\lim_{x \to 3^-} f(x) = \infty$.

• Oblicua $\boxed{y = x}$ cuando $x \to -\infty$.

$m = \lim_{x \to -\infty} \dfrac{f(x)}{x} = \lim_{x \to -\infty} \dfrac{x^3}{x^3-9x} = 1 \neq \infty$.

$n = \lim_{x \to -\infty} [f(x) - mx] = \lim_{x \to -\infty} \left[\dfrac{x^3}{x^2-9} - x\right] =$

$= \lim_{x \to -\infty} \dfrac{\cancel{x^3} - x(\cancel{x^2}-9)}{x^2-9} = \lim_{x \to -\infty} \dfrac{+9x}{x^2-9} = 0$

• Horizontal: No tiene

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x^2-4 = \infty \to$ No tiene

$x \to +\infty$

$\boxed{A4}$

	E	$\bar{E}$	
D	$6/25$	$32/75$	$2/3$
$\bar{D}$	$4/25$	$13/75$	$1/3$
	$2/5$	$3/5$	

$E = $ " "

$D = $ " "

$P(E|D) = \frac{9}{25}$

$\frac{x}{2/3} = 9/25$

$x = \frac{2}{3} \cdot \frac{9}{25} = \frac{6}{25}$

ⓐ $P(E \cap D) \overset{?}{=} P(E) \cdot P(D)$

$\frac{6}{25} \neq \frac{2}{5} \cdot \frac{2}{3} = \frac{4}{15}$

$\hookrightarrow$ No independientes.

ⓑ $P(\bar{E} \cap \bar{D}) = \frac{13}{75}$

$\boxed{A5}$ $X =$ "peso..." $\sim N(\mu, 25)$

ⓐ $n = 15$

$\bar{x} = 560$

$\alpha = 0'05$

$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) = N\left(\mu, \frac{25}{\sqrt{15}}\right) =$

$= N(\mu, 6'45)$

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 1'96 \cdot 6'45 = 12'65$

$IC = (\bar{x} - E, \bar{x} + E) = (560 - 12'65, 560 + 12'65)$

$$IC = (\bar{x} - E, \bar{x} + E) = (560 - 12'65, 560 + 12'65) = \boxed{(547'35, 572'65)}$$

$$\boxed{P(Z \le z_{\alpha/2}) = 1 - \frac{\alpha}{2} \qquad 1 - \frac{0'05}{2} = 0'975 \longrightarrow z_{\alpha/2} = 1'96}$$

(b) $\mu = 560 \longrightarrow \bar{X}_{50} \equiv N\left(560, \frac{25}{\sqrt{50}}\right) \equiv N(560, 3'54)$

$$P(\bar{X} \ge 565) = P\left(Z \ge \frac{565 - 560}{3'54}\right) =$$

$$= P(Z \ge +1'41) =$$

$$= 1 - P(Z \le 1'41) = 1 - 0'9207 = \boxed{0'0793}$$

B1

	€
Ancho (x)	1000
Largo (y)	500
	9000

$x \ge 2$

$y \ge 5$

$y \ge 2x$

$y \le 3x$

Max x

s.a. $\cancel{1000}x + \cancel{500}y \le 9\cancel{000}$ (1)

$x \ge 2$

$y \ge 5$

$y \ge 2x$ (2)

$y \le 3x$ (3)

(1)

x	y
0	18
9	0

(2)

x	y
0	0
4	8

(3)

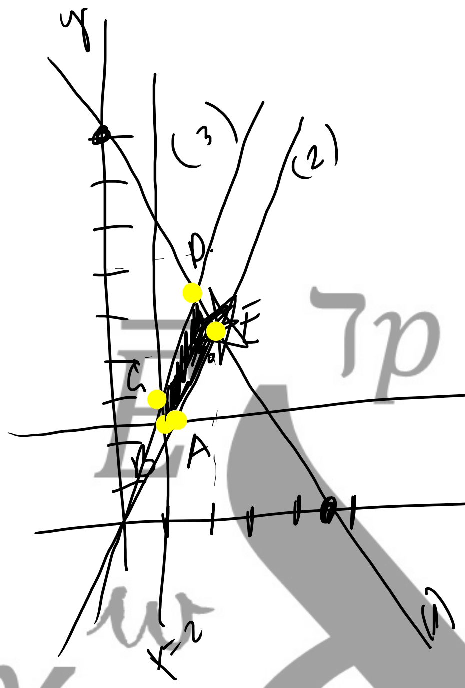
x	y

$y \geq 2x$
$y \leq 3x$ $\xrightarrow{(3)}$
$x, y \geq 0$

≥ 0

x	y
0	0
4	12

(a)



$$\left.\begin{array}{l} 10x + 5y = 90 \\ y = 3x \end{array}\right\} \quad 10x + 15x = 90 \quad 25x = 90 \quad x = \frac{18}{5} = 3'6$$

$$\left.\begin{array}{l} 10x + 5y = 90 \\ y = 2x \end{array}\right\} \quad 10x + 10x = 90 \quad x = \frac{9}{2}$$

A (2'5, 5)
B (2, 5)
C (2, 6)
D (3'6, 10'8)
$\boxed{E (4'5, 9)}$

Ancho 4'5
Largo 9
Coste 9000 €

B2

$$A = \begin{pmatrix} 3 & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$$

$$(AB)^{-1} = \frac{1}{2} \begin{pmatrix} 0 & 3 & -1 \\ 0 & -1 & 1 \\ 2 & -3 & -3 \end{pmatrix}$$

$$(A\cdot B)^{-1} = \frac{1}{2}\begin{pmatrix} 0 & \cdots & \cdots \\ 2 & \cdots & -3 \end{pmatrix}$$

ⓐ $A^{-1} = \frac{1}{|A|}\left(Adj\ A\right)^t$ $\qquad \begin{pmatrix} 3 & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$

$$|A| = \cancel{18} + 60 + 64 - 40 - 96 - \cancel{18} = -12$$

$$(Adj\ A) = \begin{pmatrix} 0 & -4 & 2 \\ -18 & -22 & 23 \\ 6 & +14 & -13 \end{pmatrix}$$

$$\boxed{A^{-1} = \frac{1}{12}\begin{pmatrix} 0 & 18 & -6 \\ 4 & 22 & -14 \\ -2 & -23 & 13 \end{pmatrix}}$$

ⓑ B^{-1} ?

$$(A\cdot B)^{-1}\cdot A = B^{-1}\cdot \cancel{A^{-1}\cdot A} = \frac{1}{2}\begin{pmatrix} 0 & 3 & -1 \\ 0 & -1 & 1 \\ 2 & -3 & -3 \end{pmatrix} A$$

$$B^{-1} = \frac{1}{2}\begin{pmatrix} 0 & 3 & -1 \\ 0 & -1 & 1 \\ 2 & -3 & -3 \end{pmatrix}\begin{pmatrix} 3 & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix} =$$

$$= \frac{1}{2}\begin{pmatrix} 2 & 0 & 0 \\ 2 & 2 & 4 \\ -12 & 4 & -4 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ -6 & 2 & -2 \end{pmatrix}}$$

B3 $f(x) = x^3 + x^2 - 5x + 3$

(a)

	1	1	−5	3
1		1	2	−3
	1	2	−3	$\boxed{0}$
1		1	3	
	1	3	$\boxed{0}$	
−3		−3		
	1	$\boxed{0}$		

Corte OX: $(1,0)$, $(-3,0)$

Corte OY: $(0,3)$

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

(b) ¿$f'(x) = 3$?

$$f'(x) = 3x^2 + 2x - 5$$

$$3x^2 + 2x - 5 = 3 \longrightarrow 3x^2 + 2x - 8 = 0 \begin{cases} x = -2 \\ x = \frac{4}{3} \end{cases}$$

$$\boxed{\text{Solución: } \begin{cases} x_1 = -2 \\ x_2 = \frac{4}{3} \end{cases}}$$

B4 $P(A) = 0'3$

B4 $P(A) = 0'3$

$P(B|A) = 0'4$

$P(B|\bar{A}) = 0'6$

ⓐ $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{0'12}{0'54} = \dfrac{2}{9} \simeq \boxed{0'22}$

$P(B|A) = \dfrac{P(A\cap B)}{P(A)} \to P(A\cap B) = P(B|A)\cdot P(A) = 0'4\cdot 0'3 = 0'12$

$P(B) = P(B|A)\cdot P(A) + P(B|\bar{A})\cdot P(\bar{A}) = 0'4\cdot 0'3 + 0'6\cdot 0'7 = 0'54$

ⓑ $P(\bar{A}|\bar{B}) = \dfrac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} = \dfrac{P(\overline{A\cup B})}{0'46} = \dfrac{1-P(A\cup B)}{0'46} = \dfrac{0'28}{0'46} = \dfrac{14}{23} \simeq \boxed{0'61}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0'3 + 0'54 - 0'12 = 0'72$

B5

ⓐ $p = 0'22$

$\hat{p} \sim N\left(p, \sqrt{\dfrac{pq}{n}}\right)$

¿n?

$\alpha = 0'01$

$E = z_{\alpha/2}\cdot\sqrt{\dfrac{pq}{n}}$

$\alpha = 0'01$

$E < 0'04$

$E = z_{\alpha/2} \cdot \sqrt{\frac{pq}{n}}$

$0'04 = 2'575 \cdot \sqrt{\frac{0'22 \cdot 0'78}{n}}$

$P(z \leq z_{\alpha/2}) = 1 - \frac{\alpha}{2}$

$\sqrt{n} = \frac{2'575 \cdot \sqrt{0'22 \cdot 0'78}}{0'04}$

$1 - \frac{0'01}{2} = 0'995 \longrightarrow z_{\alpha/2} = 2'575$

$\sqrt{n} = 26'67$

$n = 711'1$

$\boxed{\text{Solución: } 712}$

b)

$n = 1000$

$\hat{p} = \frac{250}{1000} = 0'25$

$\alpha = 0'05$

$P \sim N\left(0'25, \sqrt{\frac{0'25 \cdot 0'75}{1000}}\right) =$

$N(0'25, 0'019)$

$E = z_{\alpha/2} \sqrt{\frac{pq}{n}} = 1'96 \cdot 0'019 =$

$= 0'038$

$P(z \leq z_{\alpha/2}) = 1 - \frac{\alpha}{2}$

$1 - \frac{\alpha}{2} = 1 - \frac{0'05}{2} = 0'975 \longrightarrow \boxed{z_{\alpha/2} = 1'96}$

$$IC_{95\%} = (\hat{p} - E,\ \hat{p} + E) =$$

$$= (0'25 - 0'038,\ 0'25 + 0'038) =$$

$$\boxed{= (0'212,\ 0'288)}$$