$\boxed{A1)}$ $(A|B) = \left(\begin{array}{ccc|c} 3 & 2 & a & 1 \\ a & 1 & -1 & 2 \\ 5 & 3 & 1 & 2a \end{array}\right)$

$|A| = 3 + 3a^2 - 10 - 5a - 2a + 9 =$

$= 3a^2 - 7a + 2$

$|A| = 0 \Leftrightarrow 3a^2 - 7a + 2 = 0 \leftrightarrow \boxed{\begin{array}{l} a = 2 \\ a = 1/3 \end{array}}$

• Si $a \neq 2, 1/3 \rightarrow |A| \neq 0 \rightarrow \left.\begin{array}{l} rg(A) = 3 \\ rg(A|B) = 3 \\ n^o \text{ incog} = 3 \end{array}\right\}$ Th. R-F S.C.D.

• Si $a = 2$ $\left(\begin{array}{ccc|c} 3 & 2 & 2 & 1 \\ 2 & 1 & -1 & 2 \\ 5 & 3 & 1 & 4 \end{array}\right)$ $\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = -1 \neq 0$

$|A| = 0$ $rg(A) = 2$

$rg(A) < 2$

$\begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 5 & 3 & 4 \end{vmatrix} = 12 + \cancel{6} + \cancel{20} - \cancel{5} - \cancel{16} - 18$

$= 17 - 18 = -1 \neq 0$

$rg(A|B) = 3$

$\boxed{rg(A) \neq rg(A|B) \longrightarrow SI}$

• Si $a = 1/3$ $\left(\begin{array}{ccc|c} 3 & 2 & 1/3 & 1 \\ 1/3 & 1 & -1 & 2 \\ 5 & 3 & 1 & 2/3 \end{array}\right)$ $\begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = 4 \neq 0$

$|A| = 0$ $rg(A) = 2$

$rg(A) < 2$

$|A|=0$

$rg(A) \leq 2$

$$\left(\begin{array}{ccc|c} 1/3 & 1 & -1 & 2 \\ 5 & 3 & 1 & 2/3 \end{array}\right) \quad rg(A)=2$$

$$\begin{vmatrix} 2 & 1/3 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 2/3 \end{vmatrix} = \frac{-4}{3} + 2 + \cancel{1} + \cancel{3} - \frac{2}{9} - \cancel{4}$$

$$= \frac{-12}{9} - \frac{2}{9} + 2 = \frac{4}{9} \neq 0$$

$rg(A|B) = 3$

$$\boxed{rg(A) \neq rg(A|B) \longrightarrow SI}$$

A2

$A(0,0,2) \quad B(2,0,1) \quad C(0,2,1) \quad D(-2,2,-1)$

a) $\overline{AB}(2,0,-1)$

$\overline{AC}(0,2,-1)$

$A(0,0,2)$

$$\pi \equiv \begin{vmatrix} x & y & z-2 \\ 0 & 2 & -1 \\ 2 & 0 & -1 \end{vmatrix} = 0$$

$\pi = x(-2) - y(2) + (z-2)(-4)$

$\pi \equiv -2x - 2y - 4z + 8 = 0$

$$\boxed{\pi \equiv x + y + 2z - 4 = 0}$$

b) $D \overset{?}{\in} \pi \qquad -2 + \cancel{2} - \cancel{2} - 4 = -6 \neq 0$

$\Longrightarrow D \notin \pi$

c)

1

c) $\boxed{A = \frac{1}{2} \left| \overrightarrow{BC} \times \overrightarrow{BD} \right| = \frac{1}{2} 4\sqrt{3} = 2\sqrt{3}\ u^2}$

$\overrightarrow{BC} = (-2, 2, 0)$

$\overrightarrow{BD} = (-4, 2, -2)$

$$\overrightarrow{BC} \times \overrightarrow{BD} = \begin{vmatrix} i & j & k \\ -2 & 2 & 0 \\ -4 & 2 & -2 \end{vmatrix} = (-4, -4, 4)$$

$\left| \overrightarrow{BC} \times \overrightarrow{BD} \right| = \sqrt{16+16+16} = \sqrt{48} = 4\sqrt{3}$

$\boxed{A3}$ $\operatorname{sen}(x^2) = x - 1 \Longleftrightarrow \operatorname{sen}(x^2) - x + 1 = 0$

$f(x) = \operatorname{sen}(x^2) - x + 1$

Th. Bolzano: ① f continua en $[a, b]$
② $f(a) \cdot f(b) < 0$

$f(0) = 0 - 0 + 1 > 0$

$f(2) = \operatorname{sen} 4 - 2 + 1 = \operatorname{sen} 4 - 1 < 0$

$\Longrightarrow$ Th Bolzano $\exists c \in (0, 2) \mid f(c) = 0.$

A4 ... $x^2 + 1$ ... $g(x) = \frac{1}{...} x^2 - 2$

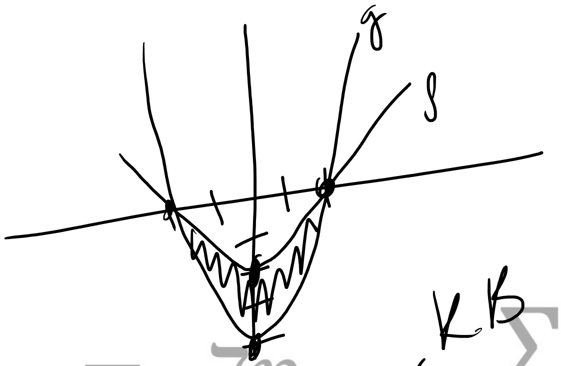
A4

$f(x) = x^2 - 4 \qquad g(x) = \frac{1}{2}x^2 - 2$

a)

x	0	2	−2
$f(x)$	−4	0	0

x	0	2	−2
$g(x)$	−2	0	0



b)

KB

$$A = \left| \int_{-2}^{2} \big(f(x) - g(x)\big)\,dx \right| = \left| H(2) - H(-2) \right| = \left| -\frac{8}{3} - \frac{8}{3} \right| = \boxed{\frac{16}{3}\,u^2}$$

$$H(x) = \int \left(x^2 - 4 - \frac{1}{2}x^2 + 2\right) dx = \int \left(\frac{1}{2}x^2 - 2\right) dx =$$

$$= \frac{1}{2}\frac{x^3}{3} - 2x$$

$$H(2) = \frac{4}{3} - 4 = \frac{-8}{3}$$

$$H(-2) = \frac{-4}{3} + 4 = \frac{8}{3}$$

A5

$A \equiv$ Chica $\qquad B \equiv$ Chico $\qquad F \equiv$ Facebook

$A \xrightarrow{8/12} F$

a) $p(A \cap F) = \frac{12}{20} \cdot \frac{8}{12} = \frac{8}{20} = \frac{2}{5} = 0{,}4$

$$12/20 \; A \begin{cases} 8/12 \; F \\ \bar{F} \end{cases} \qquad 8/20 \; B \begin{cases} 6/8 \; F^* \\ \bar{F} \end{cases}$$

ⓐ $p(A \cap F) = \frac{12}{20} \cdot \frac{8}{12} = \frac{8}{20} = \boxed{\frac{2}{5} = 0{,}4}$

ⓑ $p(B|F) = \dfrac{\frac{8}{20} \cdot \frac{6}{8}}{\frac{12}{20} \cdot \frac{8}{12} + \frac{8}{20} \cdot \frac{6}{8}} = \frac{6}{14} = \boxed{\frac{3}{7}}$

$\boxed{B1}$

$$A \begin{pmatrix} 3 & -1 & 1 \\ \lambda & 1 & \lambda \\ 0 & -\lambda & -1 \end{pmatrix}$$

ⓐ $|A| = -3 - \lambda^2 - \lambda + 3\lambda^2 = 2\lambda^2 - \lambda - 3$

$|A| = 0 \Leftrightarrow 2\lambda^2 - \lambda - 3 = 0 \rightarrow \boxed{\begin{matrix} \lambda = -1 \\ \lambda = 3/2 \end{matrix}}$

$\boxed{\text{Si } \lambda \neq -1, 3/2 \rightarrow |A| \neq 0 \rightarrow A \text{ invertible}}$

ⓑ $\lambda = 1$ $\begin{pmatrix} 3 & -1 & 1 \\ 1 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix}$ $|A| = -2$

$A^{-1} = \frac{1}{|A|} (Adj\, A)^t$ $\qquad Adj\, A = \begin{pmatrix} 0 & 1 & -1 \\ -2 & -3 & +3 \\ -2 & -2 & 4 \end{pmatrix}$

$$A^{-1} = \begin{pmatrix} 0 & 1 & 1 \\ -1/2 & 3/2 & 1 \\ [illegible] & [illegible] & [illegible] \end{pmatrix}$$

$$\boxed{A^{-1} = \begin{pmatrix} -1/2 & 3/2 & 1 \\ +1/2 & -3/2 & -2 \end{pmatrix}}$$

B2 $A(1,0,2)$ $B(3,-2,-2)$

$\overrightarrow{AB}(2,-2,-4)$

$\overrightarrow{AB}(1,-1,-2)$

$\pi \equiv x - y - 2z + D = 0$

$M = \frac{1}{2}(A+B)$

$M = (2,-1,0)$

$2 + 1 + D = 0 \rightarrow \boxed{D = -3}$

$$\boxed{\pi \equiv x - y - 2z - 3 = 0}$$

B3 $f(x) = x^2 e^x \rightarrow \mathrm{Dom}\, f = \mathbb{R}$.

$f'(x) = 2x e^x + x^2 e^x = e^x(2x + x^2)$

$f'(x) = 0 \Leftrightarrow 2x + x^2 = 0$

$x(2+x) = 0 \Rightarrow x = 0, \quad x = -2$

		-2		0	
f'	$+$		$-$		$+$
	$\nearrow$		$\searrow$		$\nearrow$

$$\boxed{\begin{array}{l} \text{Crece: } (-\infty,-2) \cup (0,+\infty) \\ \text{Decrece: } (-2,0) \end{array}}$$

$$\boxed{\begin{array}{l}\text{Crece: } (-\infty,-2)\cup(0,+\infty) \\ \text{Decrece: } (-2,0)\end{array}}$$

$\boxed{B4}$ $$\int \frac{5x+3}{x^2+2x-3}\,dx \underset{*}{=} \int \frac{3}{x+3}dx + \int \frac{2}{x-1}dx =$$

$*$ $$x^2+2x-3=0 \rightarrow x=\frac{-2\pm\sqrt{4+12}}{2}=\frac{-2\pm4}{2} \begin{cases} -3 \\ 1 \end{cases}$$

$$x^2+2x-3=(x+3)(x-1)$$

$$\frac{5x+3}{x^2+2x-3}=\frac{A}{x+3}+\frac{B}{x-1}=\frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$$

$$5x+3=A(x-1)+B(x+3)$$

$$x=-3 \longrightarrow -12=-4A \longrightarrow A=3$$
$$x=1 \longrightarrow 8=4B \longrightarrow B=2$$

$$\boxed{=3\ln|x+3|+2\ln|x-1|+C}$$

$\boxed{B5}$ $X \equiv$ "estatura ..." $\sim N(170,10)$

ⓐ $$p(170<X<185)=p\left(\frac{170-170}{10}<Z<\frac{185-170}{10}\right)=$$

$= P(0 < Z < 1'5) =$

$= P(Z < 1'5) - P(Z < 0) =$

$= 0'9332 - 0'5 = \boxed{0'4332}$

b) $P(X > a) = 0'33$

$P(X < a) = 0'67$

$P\left(Z < \frac{a-170}{10}\right) = 0'67$

$0'44 \rightarrow \frac{a-170}{10} = 0'44$

$a = 4'4 + 170 = 174'4$

$\boxed{\text{Solución: } 174'4 \text{ cm}}$