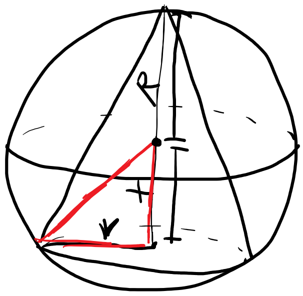
PROBLEMA 2:

Demostrar que el volumen de cualquier cono recto inscrito en una esfera es menor que el 30% del volumen de la esfera.



$$\frac{V_{cono}}{V_{esfera}} < 0'3$$

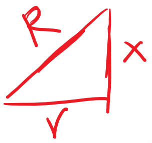
R: radio esfera
r: radio base cono
h: altura cono

$h = x + R$

$$V_{cono} = \frac{\pi \cdot r^2 \cdot h}{3}$$

$R^2 = x^2 + r^2$

$r^2 = R^2 - x^2$



$$V_{cono} = \frac{\pi}{3}\left(R^2 - x^2\right)\left(x + R\right) = \frac{\pi}{3}\left(R^2x - x^3 + R^3 - Rx^2\right)$$

$$V'_{cono} = \frac{\pi}{3}\left(R^2 - 3x^2 - 2Rx\right)$$

$R^2 - 3\frac{R^2}{16} - 2R\cdot\frac{R}{4}$

$1 - \frac{3}{16} - \frac{2}{4} = \frac{5}{16} > 0$

$$V'_{cono} = 0 \iff 3x^2 + 2Rx - R^2 = 0$$

$$x = \frac{-2R \pm \sqrt{4R^2 + 12R^2}}{2\cdot 3} = \frac{-2R \pm 4R}{6}$$

$\cancel{-R}$

$\boxed{\frac{R}{3}}$

$$V_{cono} = \frac{\pi}{3}\left(R^2 - \frac{R^2}{9}\right)\left(R + \frac{R}{3}\right) = \frac{\pi}{3}\,\frac{8R^2}{9}\cdot\frac{4R}{3} =$$

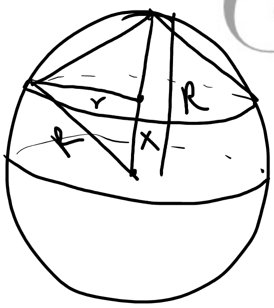
$$= \frac{32\pi R^3}{81}$$

V'_{cono}

0		$R/3$		R
	$+$		$-$	
	$\nearrow$		$\searrow$	

$\rightarrow x = \frac{R}{3}$ máximo relativo para V_{cono}.

$$\boxed{\frac{V_{cono}}{V_{esfera}} = \frac{\cancel{32\pi R^3}/81}{4\cancel{\pi R^3}/3} = \frac{32/81}{4/3} = 0'2962 < 0'3}$$



$$V_{cono} = \frac{\pi \cdot r^2 \cdot h}{3}$$

$$\boxed{h = R - x}$$

$$R^2 = x^2 + r^2$$

$$\boxed{r^2 = R^2 - x^2}$$

$$V_{cono} = \frac{\pi}{3}\left(R^2 - x^2\right)\left(R - x\right) =$$

$$= \frac{\pi}{3}\left(R^3 - R^2 x - x^2 R + x^3\right)$$

$$V'_{cono} = \frac{\pi}{3}\left(-R^2 - 2xR + 3x^2\right)$$

$$V'_{cono} = 0 \Longleftrightarrow 3x^2 - 2xR - R^2 = 0$$

$$x = \frac{2R \pm \sqrt{4R^2 + 12R^2}}{6} = \frac{2R \pm 4R}{6} \begin{cases} \cancel{R} \\ \boxed{\dfrac{-R}{3}} \end{cases}$$

$$h = R + \frac{R}{3} = \frac{4R}{3}$$