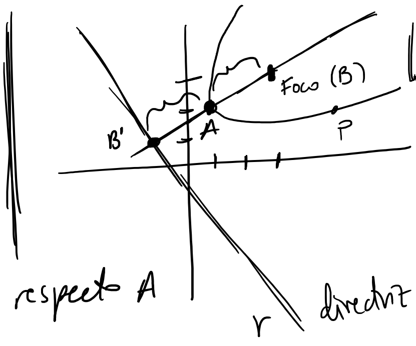
PROBLEMA Nº2

Dados los puntos del plano A(1,2) y B(3,3), se pide:

a) Calcular la ecuación de la parábola que tiene el vértice en el punto A y el foco en el punto B. (4 puntos).

b) Determinar cómo número complejo en forma binómica los vértices de un triángulo equilátero con centro en A, sabiendo que B es uno de sus vértices. (6 puntos).



$\rightarrow d(P,B) = d(P,r)$

$\boxed{d(A,B) = d(A,r)}$

(a)

— directriz — B' simétrico de B respecto A

dirección $\perp \overrightarrow{AB}$

$\overrightarrow{AB} = B - A = (2,1)$ $\quad r \equiv \begin{cases} \vec{n}\,(1,-2) \\ B'(-1,1) \end{cases}$

$A = \frac{1}{2}(B + B') \rightarrow B' = 2A - B = (2,4) - (3,3) = (-1,1)$

$r \equiv \frac{x+1}{1} = \frac{y-1}{-2} \rightarrow -2x - 2 = y - 1 \rightarrow -2x - y - 1 = 0$

$\boxed{r \equiv 2x + y + 1 = 0} \Longleftarrow$ directriz

$P(x,y) \rightarrow d(P,B) = d(P,r)$

$\overrightarrow{PB} = B - P = (3-x, 3-y) \rightarrow d(P,B) = |\overrightarrow{PB}| = \sqrt{(3-x)^2 + (3-y)^2}$

$d(P,r) = \frac{|2x+y+1|}{\sqrt{2^2+1^2}} = \frac{|2x+y+1|}{\sqrt{5}}$

$\left(\sqrt{(3-x)^2 + (3-y)^2}\right)^2 = \left(\frac{|2x+y+1|}{\sqrt{5}}\right)^2 \rightarrow$

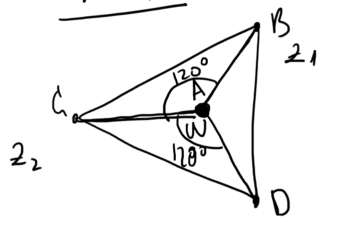
$\rightarrow 9 - 6x + x^2 + 9 - 6y + y^2 = (2x+y+1)^2 \cdot \frac{1}{5}$

$(2x+y+1)(2x+y+1) =$
$4x^2 + 2xy + 2x$
$2xy + y^2 + y$
$2x + y + 1$
$4x^2 + 4xy + 4x + y^2 + 2y + 1$

$90 - 30x + 5x^2 - 30y + 5y^2 = 4x^2 + 4xy + 4x + y^2 + 2y + 1$

$\boxed{x^2 + 4y^2 - 4xy - 34x - 32y + 89 = 0}$

(b) A(1,2) B(3,3)



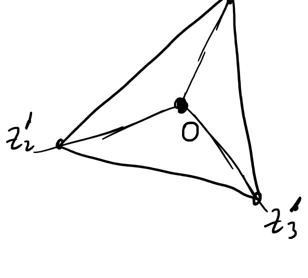
$\boxed{w = 1 + 2i}$

$z_1 = 3 + 3i$

$\Longrightarrow$

$z'_1 = z_1 - w$

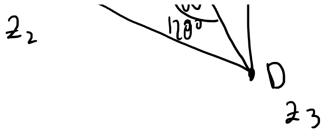
$z'_2 = z_2 - w$



$\boxed{z'_1 = 2 + i}$

$\begin{cases} |z'_1| = \sqrt{2^2+1^2} = \sqrt{5} \\ tg\,\theta = \frac{1}{2} \rightarrow \theta = arctg\left(\frac{1}{2}\right) \end{cases}$

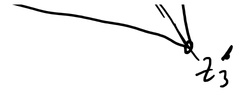
$\Downarrow$



$z_1' = z_1 - w$

$z_2' = z_2 - w$

$z_3' = z_3 - w$



$$\boxed{z_1' = \sqrt{5}_{\theta}}$$

$z_1' = \sqrt{5}_{\theta} \Longrightarrow z_1' = \sqrt{5}\,(\cos\theta + i\,\text{sen}\,\theta)$

$z_2' = \sqrt{5}_{\theta + \frac{2\pi}{3}} \Longrightarrow z_2' = \sqrt{5}\left(\cos\left(\theta + \frac{2\pi}{3}\right) + i\,\text{sen}\left(\theta + \frac{2\pi}{3}\right)\right)$

$z_3' = \sqrt{5}_{\theta + \frac{4\pi}{3}} \Longrightarrow z_3' = \sqrt{5}\left(\cos\left(\theta + \frac{4\pi}{3}\right) + i\,\text{sen}\left(\theta + \frac{4\pi}{3}\right)\right)$

$\text{tg}\,\theta = \frac{1}{2}$

$\text{tg}\,\theta = \frac{\text{sen}\,\theta}{\cos\theta} \rightarrow \frac{\text{sen}\,\theta}{\cos\theta} = \frac{1}{2} \rightarrow \boxed{2\,\text{sen}\,\theta = \cos\theta} \rightarrow \boxed{\cos\theta = \frac{2\sqrt{5}}{5}}$

$\text{sen}^2\theta + \cos^2\theta = 1 \rightarrow \text{sen}^2\theta + 4\,\text{sen}^2\theta = 1 \rightarrow 5\,\text{sen}^2\theta = 1 \rightarrow \boxed{\text{sen}\,\theta = \frac{\sqrt{5}}{5}}$

$z_1' = \sqrt{5}\left(\frac{2\sqrt{5}}{5} + i\,\frac{\sqrt{5}}{5}\right) = 2 + i$

$\text{sen}(\alpha+\beta) = \text{sen}\,\alpha\cos\beta + \text{sen}\,\beta\cos\alpha$

$\text{sen}\left(\theta + \frac{2\pi}{3}\right) = \text{sen}\,\theta\cos\frac{2\pi}{3} + \text{sen}\left(\frac{2\pi}{3}\right)\cos\theta =$

$= \frac{\sqrt{5}}{5}\cdot\left(\frac{-1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\cdot\frac{2\sqrt{5}}{5}$

$= \frac{-\sqrt{5}}{10} + \frac{\sqrt{15}}{5}$

$\text{sen}\left(\theta + \frac{4\pi}{3}\right) = \text{sen}\,\theta\cdot\cos\left(\frac{4\pi}{3}\right) + \text{sen}\left(\frac{4\pi}{3}\right)\cdot\cos\theta$

$= \frac{\sqrt{5}}{5}\left(\frac{-1}{2}\right) + \left(\frac{-\sqrt{3}}{2}\right)\frac{2\sqrt{5}}{5}$

$= \frac{-\sqrt{5}}{10} - \frac{\sqrt{15}}{5}$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \text{sen}\,\alpha\,\text{sen}\,\beta$

$\cos\left(\theta + \frac{2\pi}{3}\right) = \cos\theta\cdot\cos\frac{2\pi}{3} - \text{sen}\,\theta\,\text{sen}\,\frac{2\pi}{3} =$

$= \frac{2\sqrt{5}}{5}\left(\frac{-1}{2}\right) - \frac{\sqrt{5}}{5}\left(\frac{\sqrt{3}}{2}\right) =$

$= \frac{-\sqrt{5}}{5} - \frac{\sqrt{15}}{10}$

$\cos\left(\theta + \frac{4\pi}{3}\right) = \cos\theta\cdot\cos\left(\frac{4\pi}{3}\right) - \text{sen}\,\theta\cos\left(\frac{4\pi}{3}\right)$

$= \frac{2\sqrt{5}}{5}\left(\frac{-1}{2}\right) - \frac{\sqrt{5}}{5}\left(\frac{-\sqrt{3}}{2}\right)$

$= \frac{-\sqrt{5}}{5} + \frac{\sqrt{15}}{10}$

$z_2' = \sqrt{5}\left[\frac{-\sqrt{5}}{5} - \frac{\sqrt{15}}{10} + i\left(\frac{-\sqrt{5}}{10} + \frac{\sqrt{15}}{5}\right)\right] = \left(-1 - \frac{\sqrt{3}}{2}\right) + i\left(\frac{-1}{2} + \sqrt{3}\right)$

$z_3' = \sqrt{5}\left[\frac{-\sqrt{5}}{5} + \frac{\sqrt{15}}{10} + i\left(\frac{-\sqrt{5}}{10} - \frac{\sqrt{15}}{5}\right)\right] = \left(-1 + \frac{\sqrt{3}}{2}\right) + i\left(\frac{-1}{2} - \sqrt{3}\right)$

$w = 1 + 2i$

$$\boxed{\begin{aligned} z_1 &= 3+3i \\ z_2 &= z_2' + w = \frac{-\sqrt{3}}{2} + \left(\frac{3}{2}+\sqrt{3}\right) i \\ z_3 &= z_3' + w = \frac{\sqrt{3}}{2} + \left(\frac{3}{2}-\sqrt{3}\right) i \end{aligned}}$$