PROBLEMA 1:

Sean $f: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ una aplicación lineal con matriz asociada en la bases $\{e_i\}$ de $\mathbb{R}^4$ y $\{e'_i\}$ de $\mathbb{R}^3$:

$$A = \begin{pmatrix} 3 & 0 & 1 & 1 \\ 2 & 1 & 1 & 5 \\ 1 & 2 & 1 & 0 \end{pmatrix},$$

a) Calcula la expresión matricial de la aplicación.
b) Calcula $Ker\ f$ y una base suya.
c) Calcula $Img\ f$ y una base suya.
~~d) Escribe las bases canónicas de f respecto a $\mathbb{R}^4$ y $\mathbb{R}^3$~~

(a) $\beta = \{e_1, e_2, e_3, e_4\}$ $\beta' = \{e'_1, e'_2, e'_3\}$

$(x_1, x_2, x_3, x_4) \in \mathbb{R}^4$

$(y_1, y_2, y_3) \in \mathbb{R}^3$

$$\boxed{\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{\beta'} = \begin{pmatrix} 3 & 0 & 1 & 1 \\ 2 & 1 & 1 & 5 \\ 1 & 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}_{\beta}}$$

$Ker\ f = \{ \bar{x} \in \mathbb{R}^4 \mid A\bar{x} = 0 \}$

$Img\ f = \{ \bar{y} \in \mathbb{R}^3 : \exists\, \bar{x} \in \mathbb{R}^4 \mid A\bar{x} = \bar{y} \}$

$$\left(\begin{array}{cccc|c} 3 & & 1 & 1 & y_1 \\ 2 & & 1 & 5 & y_2 \\ 1 & & 1 & 0 & y_3 \end{array}\right) \begin{array}{c} \\ 3F_2 - 2F_1 \\ 3F_3 - F_1 \end{array} \left(\begin{array}{cccc|c} 3 & 0 & 1 & 1 & y_1 \\ 0 & 3 & 1 & 13 & 3y_2 - 2y_1 \\ 0 & 6 & 2 & -1 & 3y_3 - y_1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 2 & 1 & 1 & 5 & y_2 \\ 1 & 2 & 1 & 0 & y_3 \end{array}\right) \begin{array}{l} 3F_2-2F_1 \\ 3F_3-F_1 \end{array} \left(\begin{array}{cccc|l} 0 & 6 & 2 & -1 & 3y_3-y_1 \end{array}\right) \quad F_3-2F_2$$

$$\left(\begin{array}{cccc|l} 3 & 0 & 1 & 1 & y_1 \\ 0 & 3 & 1 & 13 & 3y_2-2y_1 \\ 0 & 0 & 0 & -27 & 3y_3-y_1-6y_2+4y_1 \end{array}\right)$$

(b)

$$\left.\begin{array}{l} 3x_1 + x_3 + x_4 = 0 \\ 3x_2 + x_3 + 13x_4 = 0 \\ -27x_4 = 0 \end{array}\right\} \longrightarrow \left.\begin{array}{l} 3x_1 + x_3 = 0 \\ 3x_2 + x_3 = 0 \\ x_4 = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} x_1 = -x_3/3 \\ x_2 = -x_3/3 \\ x_3 = x_3 \\ x_4 = 0 \end{array}\right. \quad x_3 \in \mathbb{R}$$

$$\beta_{\ker f} = \{(-1,-1,3,0)_\beta\}$$

$$\boxed{\beta_{\ker f} = \{-e_1 - e_2 + 3e_3\}}$$

(c) $\operatorname{Img} f = \mathbb{R}^3 \qquad \beta_{\operatorname{Img}} = \{e_1', e_2', e_3'\}$

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