Sea el punto P(p,q) que se encuentra en el lugar geométrico tal $x^{1/2} + y^{1/2} = a^{1/2}$. Si la recta tangente en dicho punto corta en los ejes en los puntos (0,m) y (n,0), demostrar que m+n=a.

$x = x_0$

$y = f(x)$

$$x^{1/2} + y^{1/2} = a^{1/2} \Rightarrow y - f(x_0) = f'(x_0)(x - x_0)$$

$$\Rightarrow \boxed{y - q = f'(p)(x-p)}$$

$$y^{1/2} = a^{1/2} - x^{1/2} \to y = \left(a^{1/2} - x^{1/2}\right)^2 \to \boxed{f(x) = \left(a^{1/2} - x^{1/2}\right)^2}$$

$$f'(x) = \cancel{2}\left(a^{1/2} - x^{1/2}\right)\left(-\frac{1}{\cancel{2}}x^{-1/2}\right) = -\left(a^{1/2} - x^{1/2}\right) \cdot x^{-1/2}$$

$$\left.\begin{array}{l} f'(p) = -\left(a^{1/2} - p^{1/2}\right) p^{-1/2} \\ q = f(p) = \left(a^{1/2} - p^{1/2}\right)^2 \end{array}\right\} \Rightarrow y - \left(a^{1/2} - p^{1/2}\right)^2 = -\left(a^{1/2} - p^{1/2}\right) p^{-1/2} \cdot (x-p)$$

$$\Downarrow$$

$$y = -\left(a^{1/2} - p^{1/2}\right) \cdot p^{-1/2} \cdot x + \left(a^{1/2} - p^{1/2}\right) p^{1/2} + \left(a^{1/2} - p^{1/2}\right)^2 =$$

$$= \left(a^{1/2} - p^{1/2}\right)\left[-p^{-1/2} \cdot x + \cancel{p^{1/2}} + a^{1/2} - \cancel{p^{1/2}}\right] =$$

$$= \left(a^{1/2} - p^{1/2}\right)\left[-p^{-1/2} x + a^{1/2}\right]$$

Ec. recta tangente en (p,q)

$$\boxed{y = \left(a^{1/2} - p^{1/2}\right)\left(-p^{-1/2} x + a^{1/2}\right)}$$

(0,m) (n,0)

$$y(0) = m \to \left(a^{1/2} - p^{1/2}\right) a^{1/2} = m$$

$$y(n) = 0 \to \left(a^{1/2} - p^{1/2}\right)\left(-p^{-1/2} n + a^{1/2}\right) = 0$$

- $a^{1/2} - p^{1/2} \neq 0 \Rightarrow -p^{-1/2} n + a^{1/2} = 0 \to n = +a^{1/2} p^{1/2}$

$$\boxed{n+m} = a^{1/2} \cdot p^{1/2} + \left(a^{1/2} - p^{1/2}\right) \cdot a^{1/2} = \cancel{a^{1/2} \cdot p^{1/2}} + a - \cancel{p^{1/2} \cdot a^{1/2}} = \boxed{a}$$

- $a^{1/2} - p^{1/2} = 0 \to a^{1/2} = p^{1/2} \to \boxed{a = p} \to \boxed{p = a}$, $\boxed{q = 0}$; $\boxed{m = 0}$

$\longrightarrow \boxed{m=0}$ $\boxed{y=e}$

$\longrightarrow$ recta tangente en $(a,0)$ $\boxed{y=0}$ $(0,m)$ $\boxed{(n,0)}$

que corta en infinitos puntos del tipo $(n,0) \Rightarrow \exists \infty$ valores de n