Sea el punto P(p,q) que se encuentra en el lugar geométrico tal $x^{1/2} + y^{1/2} = a^{1/2}$. Si la recta tangente en dicho punto corta en los ejes en los puntos (0,m) y (n,0), demostrar que m+n=a.

$x = x_0$

$y = f(x)$

$$x^{1/2} + y^{1/2} = a^{1/2} \Longrightarrow y - f(x_0) = f'(x_0)(x - x_0)$$

$$\boxed{y - q = f'(p)(x - p)}$$

$$y^{1/2} = a^{1/2} - x^{1/2} \longrightarrow y = (a^{1/2} - x^{1/2})^2 \longrightarrow \boxed{f(x) = (a^{1/2} - x^{1/2})^2}$$

$$f'(x) = \cancel{2}(a^{1/2} - x^{1/2})\left(-\frac{1}{\cancel{2}}x^{-1/2}\right) = -(a^{1/2} - x^{1/2}) \cdot x^{-1/2}$$

$$\left\{\begin{array}{l} f'(p) = -(a^{1/2} - p^{1/2})\,p^{-1/2} \\ q = f(p) = (a^{1/2} - p^{1/2})^2 \end{array}\right\} \Rightarrow y - (a^{1/2} - p^{1/2})^2 = -(a^{1/2} - p^{1/2})p^{-1/2} \cdot (x - p)$$

$$y = -(a^{1/2} - p^{1/2}) \cdot p^{-1/2} \cdot x + (a^{1/2} - p^{1/2})\,p^{1/2} + (a^{1/2} - p^{1/2})^2 =$$

$$= (a^{1/2} - p^{1/2})\left[-p^{-1/2} \cdot x + \cancel{p^{1/2}} + a^{1/2} - \cancel{p^{1/2}}\right] =$$

$$= (a^{1/2} - p^{1/2})\left[-p^{-1/2}x + a^{1/2}\right]$$

Ec. recta tangente en (p,q)

$$\boxed{y = (a^{1/2} - p^{1/2})(-p^{-1/2}x + a^{1/2})} \quad (0,m)\ (n,0)$$

$$\left\{\begin{array}{l} y(0) = m \longrightarrow (a^{1/2} - p^{1/2})\,a^{1/2} = m \\ y(n) = 0 \longrightarrow (a^{1/2} - p^{1/2})(-p^{-1/2}n + a^{1/2}) = 0 \end{array}\right.$$

- $a^{1/2} - p^{1/2} \neq 0 \Longrightarrow -p^{-1/2}n + a^{1/2} = 0 \longrightarrow n = +a^{1/2}p^{1/2}$

$$\boxed{n + m} = a^{1/2} \cdot p^{1/2} + (a^{1/2} - p^{1/2}) \cdot a^{1/2} =$$

$$\cancel{a^{1/2} \cdot p^{1/2}} + a - \cancel{p^{1/2} \cdot a^{1/2}} = \boxed{a}$$

- $a^{1/2} - p^{1/2} = 0 \longrightarrow a^{1/2} = p^{1/2} \longrightarrow \boxed{a = p} \longrightarrow \boxed{p = a},\ \boxed{q = 0}$

$\longrightarrow \boxed{m = 0}$

$\to \boxed{m=0}$

$\to$ recta tangente en $(a,0)$ $\boxed{y=0}$ $(0,m)$ $\boxed{(n,0)}$

que corta en infinitos puntos del tipo $(n,0) \Rightarrow \exists\ \infty$ valores de n