PROBLEMA Nº4

Consideremos la función $f(x) = \cos x$

a) Calcular la serie de Taylor de la función f. (puntos).

b) Demostrar que: $\int_0^1 \frac{\cos x}{2\cdot\sqrt{x}}dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+1)(2n)!}$ (3puntos).

c) Calcular el valor de $\int_0^1 \frac{\cos x}{2\cdot\sqrt{x}}dx$ con un error menor que 10^{-3}

th. Taylor: f k-diferenciable $x=a$

$$f(x) = \sum_{i=0}^{k} \frac{f^{(i)}(a)}{i!}(x-a)^i + R_k(x)$$

ⓐ $f(x) = \cos x = f^{IV}(x)$

$f'(x) = -\operatorname{sen} x = f^{V}(x)$ . . .

$f''(x) = -\cos x$

$f'''(x) = \operatorname{sen} x$

$$\Longrightarrow \cos x = \sum_{i=0}^{\infty} \frac{f^{(2i)}(a)}{(2i)!}(x-a)^{2i} + \sum_{i=0}^{\infty} \frac{f^{(2i+1)}(a)}{(2i+1)!}(x-a)^{2i+1} \Rightarrow$$

$$\boxed{\cos x = \sum_{i=0}^{\infty} \frac{(-1)^i \cos(a)}{(2i)!}(x-a)^{2i} + \sum_{i=0}^{\infty} \frac{(-1)^{i+1} \operatorname{sen}(a)}{(2i+1)!}(x-a)^{2i+1}}$$

$\Longrightarrow$ taylor alrededor de $x=0$

$$\boxed{\cos x = \sum_{i=0}^{\infty} \frac{(-1)^i}{(2i)!} x^{2i}}$$

$\int (af+bg)\,dx = a\int f\,dx + b\int g\,dx$

ⓑ $$\int_0^1 \frac{\cos x}{2\sqrt{x}}\,dx \overset{R.B}{=} F(1) - F(0)$$

$$F(x) = \int \frac{\cos x}{2\sqrt{x}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{x}}\cdot \cos x\,dx =$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{x}}\cdot \sum_{i=0}^{\infty} \frac{(-1)^i}{(2i)!} x^{2i}\,dx = \frac{1}{2}\left[\sum_{i=0}^{\infty} \frac{(-1)^i}{(2i)!}\int \frac{1}{\sqrt{x}}\cdot x^{2i}\,dx\right] =$$

$$= \frac{1}{2}\left[\sum_{i=0}^{\infty} \frac{(-1)^i}{(2i)!}\int x^{2i-1/2}\,dx\right] = \frac{1}{2}\sum_{i=0}^{\infty} \frac{(-1)^i}{(2i)!}\frac{x^{2i+1/2}}{2i+1/2} =$$

$$= \sum_{i=0}^{\infty} \frac{(-1)^i \cdot x^{2i+1/2}}{(2i)!\,(4i+1)}$$

$$\begin{cases} F(0) = 0 \\ F(1) = \displaystyle\sum_{i=0}^{\infty} \frac{(-1)^i}{(2i)!(4i+1)} \end{cases}$$

$$\sum_{i=0}^{\infty} \qquad I(1) = \sum_{i=0}^{\infty} \frac{(-1)^i}{(2i)!(4i+1)}$$

por lo tanto
$$\boxed{\int_0^1 \frac{\cos x}{2\sqrt{x}}\,dx = \sum_{i=0}^{\infty} \frac{(-1)^i}{(2i)!(4i+1)}}$$

ⓒ $E \leq 10^{-3} = 0'001$

$$i=0 \longrightarrow \frac{1}{1\cdot 1} = 1$$

$$i=1 \longrightarrow \frac{-1}{2!\cdot 5} = \frac{-1}{10} = -0'1$$

$$i=2 \longrightarrow \frac{1}{4!\cdot 9} = \frac{1}{216} \simeq 0'00463$$

$$i=3 \longrightarrow \frac{-1}{6!\,13} = \frac{-1}{9360} \simeq -0'000\,11$$

$\tilde{I}_0 = 1$

$\tilde{I}_1 = 0'9$

$\tilde{I}_2 = \underbrace{0'904}63$

$\tilde{I}_3 = \underbrace{0'904}52$

$$\boxed{\int_0^1 \frac{\cos x}{2\sqrt{x}}\,dx \simeq 0'904 \pm 0'001}$$