

2. Sean  $a$  y  $b$  dos números reales positivos.

a) Demostrar que si  $a < b < e$  entonces  $a^b < b^a$ .

b) Demostrar que si  $e < a < b$  entonces  $a^b > b^a$ .

$$a, b \in \mathbb{R}^{++}$$

$$\mathbb{R}^{++} = (0, +\infty)$$

$$a) \quad a^b < b^a \Leftrightarrow \ln a^b < \ln b^a \Leftrightarrow b \ln a < a \ln b$$

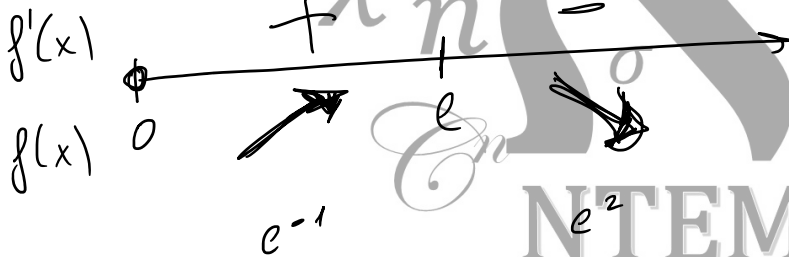
$$\Leftrightarrow \frac{\ln a}{a} < \frac{\ln b}{b}$$

$$0 < \underbrace{a < b < e} \Rightarrow \frac{\ln a}{a} < \frac{\ln b}{b} \Rightarrow f(x) = \frac{\ln x}{x} \Leftarrow$$

$$f'(x) = \frac{1/x \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$$

$$f'(x) = 0 \Leftrightarrow 1 - \ln x = 0 \Leftrightarrow \ln x = 1 \Leftrightarrow x = e$$

$$\boxed{x = e}$$



$f$  es creciente en  $(0, e) \Rightarrow$

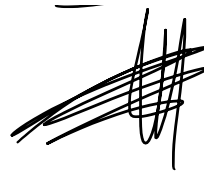
$$\forall a, b \in (0, e) \mid \boxed{a < b \Rightarrow f(a) < f(b) \Rightarrow \frac{\ln a}{a} < \frac{\ln b}{b}}$$

b)  $f$  es decreciente en  $(e, +\infty) \Rightarrow$

$$\forall a, b \in (e, +\infty) \Rightarrow \boxed{a < b \Rightarrow f(a) > f(b)}$$

$$\frac{\ln a}{a} > \frac{\ln b}{b} \Leftrightarrow$$

$$\boxed{a^b > b^a}$$



$\bar{E}$   $\tau\rho$   $\Sigma$   $K$   
 $\chi$   $w$   $n$   $\sigma$   $F$   
 $\infty$  **NTEM** **O**  
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