

3. Calcule el límite en el infinito de la sucesión A_n , siendo A_n el siguiente determinante:

$$\begin{vmatrix} 1 & -\frac{1}{2} & 0 & 0 & \dots & 0 \\ x & 1 & -\frac{1}{3} & 0 & \dots & 0 \\ x^2 & 0 & 1 & -\frac{1}{4} & \dots & 0 \\ x^3 & 0 & 0 & 1 & -\frac{1}{5} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x^{n-2} & 0 & 0 & 0 & 0 & \dots & -\frac{1}{n} \\ x^{n-1} & 0 & 0 & 0 & 0 & \dots & 1 \end{vmatrix}$$

$$A_1 = 1$$

$$A_2 = \begin{vmatrix} 1 & -1/2 \\ x & 1 \end{vmatrix} = 1 + \frac{x}{2}$$

$$A_3 = \begin{vmatrix} 1 & -1/2 & 0 \\ x & 1 & -1/3 \\ x^2 & 0 & 1 \end{vmatrix} = 1 \cdot A_2 + x^2 \begin{vmatrix} -1/2 & 0 \\ 1 & -1/3 \end{vmatrix} = 1 \cdot \left(1 + \frac{x}{2}\right) + x^2 \cdot \left(\frac{-1}{2} \cdot \frac{-1}{3}\right) = 1 + \frac{x}{2} + \frac{x^2}{6}$$

$$A_4 = \begin{vmatrix} 1 & -1/2 & 0 & 0 \\ x & 1 & -1/3 & 0 \\ x^2 & 0 & 1 & -1/4 \\ x^3 & 0 & 0 & 1 \end{vmatrix} = 1 \cdot A_3 + x^3 \cdot (-1) \begin{vmatrix} -1/2 & 0 & 0 \\ 1 & -1/3 & 0 \\ 0 & 1 & -1/4 \end{vmatrix} = 1 + \frac{x}{2} + \frac{x^2}{6} + \frac{x^3}{4!}$$

$$\begin{aligned} \dots \quad A_n &= \begin{vmatrix} 1 & -1/2 & 0 & 0 & \dots & 0 \\ x & 1 & -1/3 & 0 & \dots & 0 \\ x^2 & 0 & 1 & -1/4 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x^{n-1} & 0 & 0 & 0 & \dots & 1 \end{vmatrix} = 1 \cdot A_{n-1} + x^{n-1} \cdot (-1)^{2n} \begin{vmatrix} -1/2 & 0 & 0 & \dots & 0 \\ 1 & -1/3 & 0 & \dots & 0 \\ 0 & 1 & -1/4 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -1/n \end{vmatrix} (-1)^{n+1} \\ &= A_{n-1} + x^{n-1} \cdot \frac{-1}{n} \cdot \frac{-1}{n-1} \cdot \dots \cdot \frac{-1}{4} \cdot \frac{-1}{3} \cdot \frac{-1}{2} (-1)^{n+1} \\ &= A_{n-1} + x^{n-1} \cdot \frac{(-1)^{n-1} \cdot (-1)^{n+1}}{n!} = \\ &= A_{n-1} + \frac{x^{n-1}}{n!} \cdot (-1)^{2n} = \frac{x^{n-1}}{n!} + A_{n-1} \end{aligned}$$

$$A_n = \frac{x^{n-1}}{n!} + A_{n-1}$$

$$\dots \quad A_n = \frac{x^{n-1}}{n!} + \frac{x^{n-2}}{(n-1)!} + \dots + \frac{x^3}{4!} + \frac{x^2}{3!} + \frac{x}{2!} + 1$$

$$A_1 = 1$$

$$A_2 = 1 + \frac{x}{2}$$

$$A_3 = 1 + \frac{x}{2!} + \frac{x^2}{3!}$$

$$\Rightarrow A_1, A_2, A_3, \dots, A_n, \dots$$

$$\lim_{n \rightarrow \infty} A_n = \begin{cases} \lim_{n \rightarrow \infty} 1, & \text{si } x=0 \\ \lim_{n \rightarrow \infty} \left(\frac{x^{n-1}}{n!} + \dots + \frac{x^2}{3!} + \frac{x}{2!} + 1 \right) & \text{si } x \neq 0 \end{cases}$$

$$\lim_{n \rightarrow \infty} A_n = \begin{cases} \lim_{n \rightarrow \infty} \left(\frac{x^{n-1}}{1!} + \dots + \frac{x^2}{2!} + \frac{x}{1!} + 1 \right) & \text{si } x \neq 0 \end{cases}$$

$$\lim_{n \rightarrow \infty} \left(\frac{x^{n-1}}{1!} + \dots + \frac{x^2}{2!} + \frac{x}{1!} + 1 \right) = \sum_{k=1}^{\infty} \frac{x^{k-1}}{k!} =$$

$$= \sum_{k=1}^{\infty} \frac{x^k}{k! \cdot x} =$$

$$= \frac{1}{x} \left[\sum_{k=0}^{\infty} \frac{x^k}{k!} - 1 \right] = \frac{1}{x} [e^x - 1]$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\lim_{n \rightarrow \infty} A_n = \begin{cases} 1 & \text{si } x = 0 \\ \frac{e^x - 1}{x} & \text{si } x \neq 0 \end{cases}$$

